بسم الله الرحمن الرحيم

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DESIGN OF CONTROLLABLE / OBSERVABLE CONTROL SYSTEMS

A dissertation submitted in partial fulfillment of registration for the award of the M.Sc. Degree

By
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و الشمس ى ستقر ا ذلك تقدير العزيز العليم ⋆ و القمر قدرناه منازل حـ عاد كالعرجون القد ⋆ لا الشمس ينبغى ا أن تدرك القمر و لا الليل سابق النهار وكل فلك يسبحون ⋆

صدق ا العظيم

سورة يس من " 38-40"

اهدى مشروعى هذا لكل من

الوالد العزيز

الوالدة العزيزة

الزوجة العزيزة

يع الأخوان و الأخوات

وكل باحث   هذا الال أينما وجد ......

Dedication

This work was dedicated to

My father

My mother

My wife

My brothers and sisters
To whom I owe much …….

I.H. Ibrahim

2003

الخلاصة

تصميم التحكمية و الملاحظية لآنظمة التحكم عمليا تتم عن طريق المحددة التحكميية (S) التى تعتمد على (A,B) حيثA محددة النظام وB ترمز لمحددة تغذية النظام التحكم كما أن الملاحظية V)) التى تعتمد على A,C)) حيث C محددة الناتج. اذا كانت قيمة المجددة التتحكمية (S) و المحددة الملاحظية ذات قيمة موجبة او سالبة ،فأن النظام يكون تحكمى و ملاحظ أما

'□Hلشمس□*جرى□Dمستقر□Dها□0لك□*قدير□'لعزيز□'لعليم□
'□H□mلقمر□Bدرناه□Eنازل□-
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□'لقمر□D□Hا□'لليل□3ابق□'لنهار□Hكل□Aى□Aلك□Jسبحون□□m

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3 ورة□Jس□Eن□" 38-40"

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Eشروعى□Gذا□Dكل□Eن□'لوالد□'لعزيز□'لوالدة□'لعزيزة□'لزوجة□'لعزيزة□,ميع□'لأخوان□'□Hلأخوات□Hكل□(احث□Aى□
Gذا□'لال□#ينما□Hجد□......

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My father
My mother
My wife
My brothers and sisters
To whom I owe much &…

I.H. Ibrahim

2003

الخلاصة

* صـميم التحكميـة H لاحظيـة D آنظمـة الـتحكم 9مليـا * 9ن 7ريـق ـددة التحكمييـة (S)

الـ *عتمد 9 (A,B) -يثف E حددة النظام H ق *رمز D ددة *غذية النظـام الـتحكم C مـا #ن لاحظيـة V))

الـ *عتمـد 9 (A,C)) -يـث E C حـددة النـا خ ذا C انـت B ـة لـددة التحكميـة (S)

H ددة لاحظية 0ات B ة E وجبة و 3البة A ، أن النظام J كون *حكمى E H لاحظ #ما

{n{`J6

اذا كانت القيمة ات ا ددت صفرية فان النظام يكون غ كمى و غ ملاحظ.

هذه الدراسة استخلصت أن لقيمة ددة النظام تأ ا بالغا تصميم التحكمية و ا لاحظية لآنظمة التحكم. أذا كانت ـددة النظـام (A) موجبة أو سالبة القيمة فأن النظام يكون كمى اذا ذود غذى اى مدخل للنظام ،كما أن النظام يكون ملاحظأ مهما كانت نقطة التوصيل للناتج. و كـن س هات الصفت باضافة مغذى آخر للتحكمية او ناتج آخر للملاجظية. اما اذا كانت قيمة ددة النظام مساوية للصفر فان النظام يكون غ كمى و غ ملاحظ لبعض ا الات التوصيل و الناتج. و لكن كن ارجاعها ا الصفت باضافة مغذى آخر للتحكمي و ناتج آخر للملاحظ.

كما ان التغذية ا لفية ذات تأث موجب للتحكمية أى سن من التحكمية كما أن تأث ها سالبا للملاحظ أى أن يل النظام لغ ملاحظى.

هذه النتيجة ال استخلصت للمشروع ت باستخدام MATLAB لتسهيل العمليات الرياضية.

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Chapter II: State variable models

Chapter III: Design of controllable/ observable control systems

Chapter 1: Systems and Control Systems

1.Introduction

The controllable/ observable control systems are designed using the state equation model. In project we are interested in controlling the system with control signal u (t), which is a function of several measure of state variables. The basic ingredients of a control system can be described by objectives of control, control system components and results or outputs. In more technical term, the objectives can be identified with inputs, or actuating signals, u, and outputs or the controlled variables, y.

An important first step in the design of controllable/ observable control systems is the mathematical modeling of the controlled process. In general, given a controlled process, the set of variables that identify the dynamic characteristics of the process should first be defined. Therefore, the key words to the design linear controllable/ observable control systems are assumptions, identification, linearization and modeling.

The studies of design of controllable/ observable control systems rely to a great extent on the use of applied mathematics. One of the major purposes of the studies is to develop a set of analytical tools, so that the designer can arrive at reasonable predicable and reliable designs without depending completely on the drudgery of experimentation or extensive computer simulation. MATLAB has been used for solving differential equations

In the modern control theory, it is often desirable to use matrix notation to simply complex mathematical expressions. The matrix notation usually makes the equations much easier to handle and manipulate.

Because of their simplicity and versatility, block diagrams are often used to model all types of systems. A block diagram can be used simply to describe the composition and interconnection of system. It can be used, together with transfer function, to describe the cause-and-effect relationships through the system.

The signal-flow graph (SFG) may be regarded as a simplified version of a block diagram. The (SFG) for the cause and effect representation of linear systems, that are modeled by algebraic equations. A SFG may be defined as a graphical means of portraying the input-output relationships between the variables of a set of linear algebraic equations

The states diagram an extension of the SFG to portray the state equations, and differential equations. The significance of the state diagram is that it forms a close relationship among the state equations, and transfer functions.

In contrast to the transfer function approach to the design of controllable/ observable linear control systems, the state variable method is regarded as modern, since it is the underlying force for optimal control. Transfer functions are defined only for linear time-invariant systems. The relationship between the conventional transfer function approach and state variable approach is established

So that, the analyst will be able to investigate a system problem with various alternative methods.

Finally, the designs of controllable/ observable linear systems are defined and their applications investigated.

2-Systems

The term system has become widely used today, and as a result, its original meaning has been somewhat diluted. A system is a combination of elements intended to act together to accomplish an objective. [3]

2.1-System Classification:

System

Lumped parameter	Distributed Parameter
Linear (Super position)	Non-linear

Hybrid

Continuous	Discrete
(Differential equation)	(Difference equation)
(Laplace transform)	(z transform)
(Transfer functions-S Domain)	(Transfer function–Z Domain)
Time invariant	Time – varying
(constant parameter)	(varying parameter)
(Single-input, single-output) SISO	(multi-input, multioutput) MIMO

Signal

Deterministic	Stochastic [10]

2.2 -Static and Dynamic Systems

In general the present value of an element's output is the result of what has happened to the element in the past as well as what is currently affecting it.

A dynamic element is defined to be one whose present output depends on past inputs. Conversely; a static element is one whose output at any given time depends on only the input at that time.

In popular usage, the terms static and dynamic distinguish situations in which no change occurs from those that are subject to changes over time. This usage conforms to the preceding definitions of these terms if the proper interpretation is made. A static element's output can change with time only if input changes and will not change if the input is constant or absent. However, if the input is constant or removed from a dynamic element, its output can still change.

A static system contains all static elements. Any system that contains at least one dynamic element is a dynamic system. [3]

2.3- Types of Models

2.3.1- Lumped and distributed parameter models

Many variables in nature are functions of location as well as time. The process of ignoring the spatial dependence by choosing a single representative value is called lumping.

Lumping an element is a technique usually requiring experience. It reflects the judgment of the engineer about what is unimportant in terms of spatial variation. It can be described as the spatial equivalent of the process of dividing a system into static and dynamic elements. The model of lumped element or system is called a lumped parameter model. If it is dynamic, the only independent variable in the model will be time, that is; the model will be an ordinary differential equation. Only time derivatives will appear, not spatial derivatives.

When spatial dependence is included, the independent variables are the spatial coordinates as well as time. The resulting model is said to be a distributed parameter model. It consists of one or more partial differential equations containing partial derivatives with respect to the independent variables. The difference is illustrated in figure 1.1, which shows the temperature T of a metal plate. If the plate is heated at one side, the temperature will be function of location and time. [4]

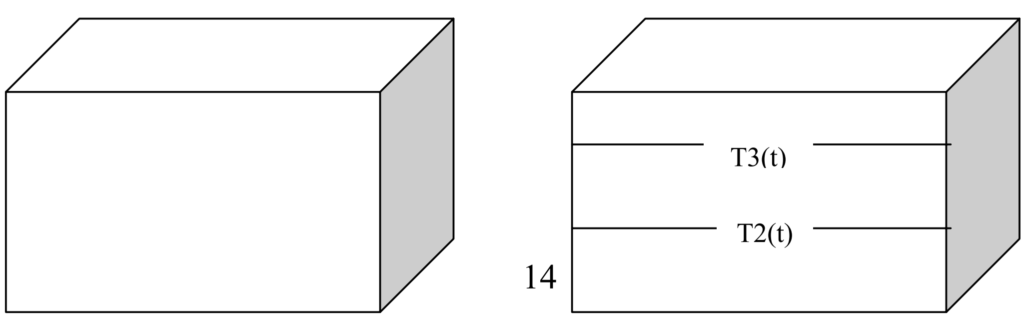


Figure 1.1 *Temperature distribution in a plate*

a) Distributed-parameter representation

b) Lumped-parameter representation using three elements

$$\mathbf{T=T(t,x,y,z)} \qquad \mathbf{1.1.1}$$

and the model will have the form.

$$\mathbf{f\,(T,\ \partial T/\partial t,\ \partial^{2}T/\partial x^{2},\ \partial^{2}T/\partial y^{2},\ \partial^{2}T/\ \partial z^{2}) = 0} \qquad \mathbf{1.1.2}$$

But if the plate temperature is lumped with a single value, the model will have the form figure (1.1.b), which is mathematically easier to handle. Lumping may be done at several levels [3]

$$\mathbf{f(T,\ dT/dt)=0} \qquad \mathbf{1.1.3}$$

2.3.2- Linear and nonlinear models

Engineers should attempt to model elements as static rather than dynamic and as lumped rather than distributed. The reason is that engineers eventually have to analyze the resulting system model, and its complexity can easily get out of hand if there is too much detail in each element model.

Let y be the output and x the input of an element that can be either static or dynamic. Its model is written as:

$$\mathbf{y = f(x)} \qquad \mathbf{1.1.4}$$

where the function f(x) may include such operations as differentiation and integration. The model is said to be linear if, for an input $ax_1 + bx_2$, the output is

$$y = f(ax_1 + bx_2) = a f(x_1) + b f(x_2) = a y_1 + b y_2 \qquad \textbf{1.1.5}$$

where a and b are arbitrary constants, x_1 and x_2 are arbitrary inputs and

$$y_1 = f(x_1) \qquad \textbf{1.1.6}$$

$$y_2 = f(x_2) \qquad \textbf{1.1.7}$$

Thus, linearity implies that multiplicative constants and additive operations in the input can be factored out when considering the effects on the output. It is sometimes called superposition principle. The definition of linearity eq. (1.1.5) can be extended to include functions of more than one variable, such as f (x, z). This function is linear if and only if $f(ax_1 + bx_2, az_1 + bz_2) = a f(x_1, z_1) + b f(x_2, z_2)$. Differential equations represent input-output relations also can be classified as linear or nonlinear.

A differential equation is easily recognized as nonlinear if it contains powers or transcendental functions of the dependent variable. For example, the following equation is nonlinear [3]

$$dy/dt = \mathrm{sqrt}(y) + f \qquad \textbf{1.1.8}$$

2.3.3-Time Variant Models

The presence of a time-varying coefficient does not make a model nonlinear. Models with constant coefficients are called time- invariant or stationary models, while those with variable coefficients are time- variant or nonstationary. [3]

$$dy/dt = c(t)y + f \qquad \textbf{1.1.9}$$

2.3.4- Discrete-and Continuous -Time Models

Sometimes it is inconvenient to view the system's dynamics in terms of a continuous-time variable. In such cases, we use discrete variable to measure time. The most important situation suggesting the use of discrete time models occurs when a system contains a digital computer for measurement or control purposes. Thus, a digital computer cannot take measurements continuously but must "sample" the measured variable at these instants.

If we choose to represent our system in terms of discrete time, the form of the model is a difference equation instead of a differential equation.[3]

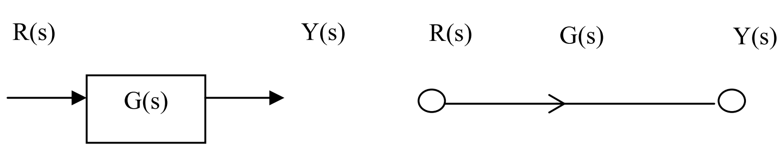
3- Control systems

3.1- Open and Closed Loop Control Systems

A control system is an interconnection of components forming a system configuration that will provide a desired system response. Because a desired system response is known a signal proportional to the error between the desired and actual response is generated. The utilization of this signal to control the process results in a closed –loop sequence of operations that is called a feedback system.

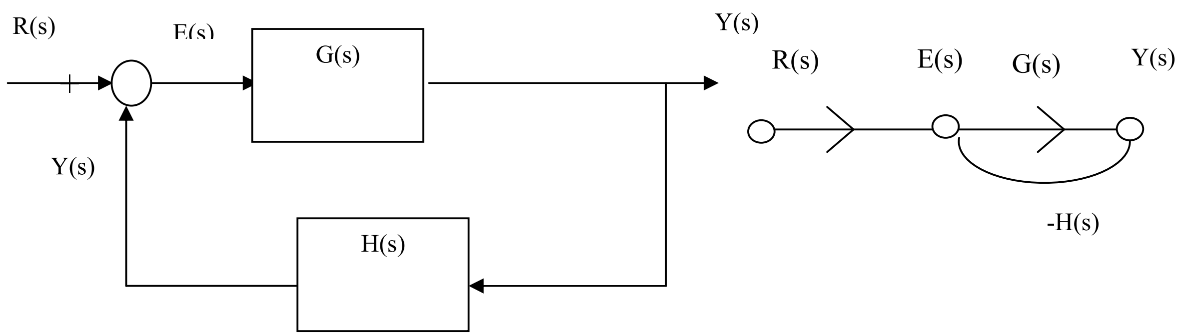
A system without feedback is often called a direct system or an open-loop system. An open-loop system operates without feedback and directly generates the output in response to an input signal. Fig 1.1.2.

A closed loop system uses a measurement of the output signal and a comparison with the desired output to generate an error signal that is applied to the actuator [5].



(a) (b)

Fig 1.1.2: *A direct system, without feedback*



(a) **(b)**

Fig.1.2.2: *A closed-loop control system (a feedback system)*

In many cases, H(s) is equal to 1 or constant other than 1. The constant accounts for a unit conversion such as radian to volts, if the unity-feedback condition with H(s) =1,then [4]

$$\mathbf{Y(s) = G(s).\ E(s) = G(s)\ [R(s) - Y(s)]} \qquad \mathbf{1.2.2}$$

$$\mathbf{Y(s) + G(s).\ Y(s) = G(s)\ R(s)}$$

$$\mathbf{Y(s) = \frac{G(s).\ R(s)}{1 + G(s)}} \qquad \mathbf{1.2.3}$$

The error signal is:

$$\mathbf{E(s) = \frac{1}{1+G(s)}\ R(s)} \qquad \mathbf{1.2.4}$$

Thus, to reduce the error, the magnitude of [1+G(s)] must be greater than one over the range of s under consideration. If $H(s) \neq 1$ the output of the closed loop system is

$$\mathbf{Y(s)=G(s).\,E_{a.}(s)=G(s)\,[R(s)-H(s)\,Y(s)]}$$

$$\mathbf{Y(s)\,(1+H(s)\,G(s)) = G(s)\,R(s)}$$

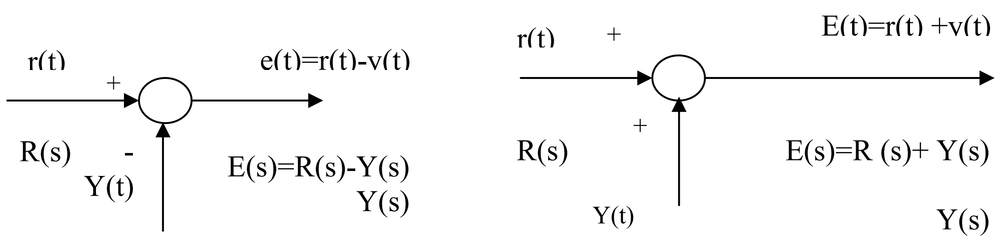
$$\mathbf{Y(s) = \frac{G(s).}{1+G.H(s)}\;R(s)} \qquad \mathbf{1.2}$$

The actuating error signal is

$$\mathbf{E_a\,(s) = \frac{1}{1+G.H(s)}\;.\;R(s)} \qquad \mathbf{1.2}$$

3.2-Block Diagrams of Control Systems

One of the important components of a control system is the sensing device that acts as junction point for signal comparisons. The physical components involved are the potentiometer; synchro, resolver, differential amplifier, multiplier, and other signal processing transducers. In general, sensing devices perform other simple mathematical operations such as addition, subtraction, multiplication (nonlinear) and sometimes combinations of these. The block diagram representations of these operations are illustrated in Figure 1.2.2.1. [1].



r2 (t)

-

$R_2(s)$

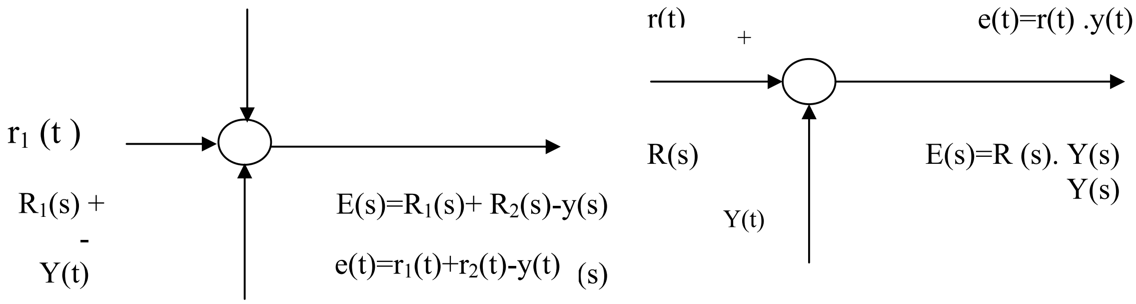


Figure 1.2.2.1: *The block diagram representation of operations*

Chapter II State variable models

1- The state variables of a dynamic system.

The designs of control systems utilize the concept of the state of a system. The state of a system is a set of numbers such that knowledge of these numbers and the input functions will, with the equations describing the dynamics, provide the future state and output of the system. For a dynamic system, the state of a system is described in terms of a set of state variables

$$[x_1(t), x_2(t), \ldots\ldots, x_n(t)] \qquad \mathbf{2.1.1}$$

The state variables are those variables that determine the future behavior of a system when the present state of the system and the excitation signals are known. Consider the system shown in fig2.1.1, where u_1 (t) and u_2 (t) are the input signals and y_1 (t),y_2 (t) are the output signals. A set of state variables [x_1, x_2, x_3,…….x_n] for the system shown in the figure 2.1.1 is a set such that knowledge of the initial values of the state variables [$x_1(t_0)$,$x_2(t_0)$,……$x_n(t_0)$] at the initial time t_o, and of the input signals u_1(t) and u_2(t) for $t \geq t_0$ suffices to determine the future values of the outputs and state variables.[2]

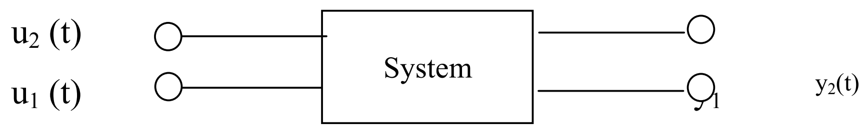


Figure 2.1.1-*system block diagram*

Then the state variables describe the future response of a system, given the present state, the excitation inputs, and the equations describing the dynamics. The general form of a dynamic system is shown in figure 2.1.2

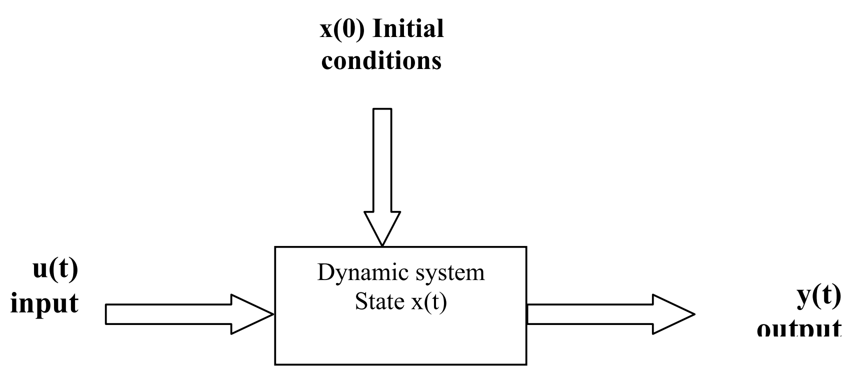


Fig2.1.2 *The general form of dynamic system*

An alternative approach to developing a model of a device is the use of the bond graph. Bond graphs can be used for electrical, mechanical, hydraulic and thermal devices or systems as well as for combinations of various types of elements. Bond graphs produce a set of equations in the state variable form.

The state variables of a system characterize the dynamic behavior of a system. The engineer's interest is primarily in physical systems, where the variables are voltages, currents, velocities, positions and similar physical variables. However, the concept of system state is not limited to the analysis of physical systems and is particularly useful for analyzing biological, social and economic systems as well as physical systems. For these system, the concept of state is extended beyond the concept of energy of a physical system to the broader viewpoint of variables that describe the future behavior of the system. [2]

2- The state differential equation

The state of the system is described by the set of first-order differential equations written in terms of the state variables (x_1,x_2,x_3, ……x_n). These first-order differential equations can be written in general form as:

$$\mathbf{x_1= a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n+b_{11}u_1+b_{12}u_2+\ldots+b_{1m}u_m}$$

$$\mathbf{x_2= a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n+b_{21}u_1+b_{22}u_2+\ldots+b_{2m}u_m}$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

2.2.1

$$\mathbf{x_{1n}= a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n+b_{n1}u_1+b_{n2}u_2+\ldots+b_{nm}u_m}$$

where x = d x/dt. Thus, this set of simultaneous differential equations can be written in matrix form as follows:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{mn} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \qquad \mathbf{2.2.2}$$

The column matrix consisting of the state variables is called the state vector and is written as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x_1} \\ \vdots \\ \mathbf{x_n} \end{bmatrix} \quad \textbf{2.2.3}$$

where boldface indicates a matrix. The matrix of input signals is defined as u.. Then the compact notation of the state differential equation as can represent the system:

$$\mathbf{x} = \mathbf{Ax} + \mathbf{Bu} \quad \textbf{2.2.4}$$

The differential equation 2.2.4 is also called the state equation

The matrix A is an n × n square matrix and B is an n × m matrix. The state differential equation relates the rate of change of the state of the system to the state of the system and the input signals. In general, the outputs of a linear system can be related to the state variables and the input signals by the state equation. [7]

$$\mathbf{y} = \mathbf{Dx} + \mathbf{Hu} \quad \textbf{2.2.5}$$

where y is the set of the output signals expressed in column vector form.

The solution of the state differential equation (eq2.2.4) can be obtained in a manner similar to the approach we utilize for solving a first-order differential equation. Consider the first-order differential equation.

$$\dot{x} = ax + bu \qquad 2.2.6$$

where x(t) and u(t) are scalar functions of time. Taking the Laplace transform of (eq2.2.6) we have

$$sX(s) - x(0) = aX(s) + bU(s) \qquad 2.2.7$$

And therefore:

$$G(s) = \frac{x(0)}{sI-a} + \frac{bU(s)}{sI-a} \qquad 2.$$

The inverse Laplace transform of (eq2.2.8) results in the solution

$$x(t) = e^{at} x(0) + \int_0^t e^{a(t-\tau)} bu(\tau)\, d\tau \qquad 2.2.9$$

The solution of the state differential equation is to be similar to eq2.2.9, and to be of differential form. The matrix exponential function is defined as

$$e^{At} = \exp(At) = I + A.t + A^2 t^2/2! + \text{---} + A^k t^k/k! \qquad 2.2.10$$

which converges for all finite t and any A. Then solution of the state differential equation is found to be:

t

$$x(t) = \exp.(At)\ x\ (o) + \int_{0} \exp.[A(t-\tau)\ Bu(\tau)]\ d\tau \qquad \textbf{2.2.11}$$

Equation (2-2-11) may be obtained by taking the Laplace transform of Eq(2-2-4) and rearranging to obtain

$$X(s) = [sI-A]^{-1}\ x(0) + [s1-A]^{-1}\ BU(s) \qquad \textbf{2.2.12}$$

where: $$[sI-A]^{-1} = \acute{Ø}\ (s) \qquad \textbf{2.13}$$

$$\acute{Ø}\ (t) = \exp\ (At) \qquad \textbf{2.2.14}$$

Taking the inverse Laplace transform of Eq (2-2-12), and noting that the second term on the right-hand side involves the product Ǿ (s) BU(s). The matrix exponential function describes the unforced response of the system and is called the fundamental or state transition matrix Ǿ (t). Therefore Eq2-2-12 can be written as

$$x(t) = \acute{Ø}(t)x(0) + \int_{0}^{t} \acute{Ø}(t-\tau)\ Bu(\tau)d\tau \qquad \textbf{2.2.15}$$

The solution to the unforced system (that is, when u = 0 is simply)

$$\textbf{2.2.16} \qquad \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} Ø_{11}(t) & \cdots & Ø_{1n}(t) \\ Ø_{21}(t) & \cdots & Ø_{2n}(t) \\ \vdots & & \vdots \\ Ø_{n1}(t) & \cdots & Ø_{nn}(t) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix}$$

—

To determine the state transition matrix, all initial condition are set to 0 except for one state variable, and the output of each state variable is evaluated. That is, the term $Ø_{ij}$ (t) is the response of the ith state variable due to an initial condition on the jth state variable, when where are zero initial conditions on all the other states. [7]

3- Signal-Flow Graph State Models

The state of system describes that system's dynamic behavior where the dynamics of the system are represented by a series of first-order differential equations. Alternatively, the dynamics of the system can represented by a state differential equation as in equation (2-2-4). In either case, it is useful to develop a state flow graph model of the system and use this model to relate the state variable concept to the familiar transfer function representation.

A system can be meaningfully described by an input-output relationship, the transfer function G(s). The signal-flow graph state model can be readily derived from the transfer function of a system. However, there is more than one alternative set of state variables, and therefore there is more than one possible form for the signal-flow graph state model. In general, we can represent a transfer function as

$$\mathbf{G(s)} = \frac{\mathbf{Y(s)}}{\mathbf{U(s)}} = \frac{\mathbf{s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}}{\mathbf{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}} \qquad \mathbf{2.3.}$$

where $n \geq m$, and all the coefficients are real positive numbers. If we multiply the numerator and denominator by s^{-n} then eq 2.3.1 will be

$$G(s) = \frac{s^{-(n-m)} + b_{n-1} s^{-(n-m+1)} + --- + b_1 s^{-(n-1)} + b_0 s^{-n}}{1+a_{n-1} s^{-1}+ --- +a_1 s^{-(n-1)}+a_0 s^{-n}} \quad 2.3.2$$

Familiarity with Mason's flow graph gain formula causes us to recognize the familiar feedback factors in the denominator and the forward-path factors in the numerator. Mason's flow graph formula is written as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_k p_k \Delta_k}{\Delta} \quad 2.3.3$$

When all the feedback loops are touching and all the forward paths touch the feedback loops eq 2.3.3 reduces to [2]

$$G(s) = \frac{\sum_k p_k}{1-\sum L} = \frac{\text{Sum of the forward-path factors}}{\text{1- Sum of the feedback loop factors}} \quad 2.3.4$$

There are several flow graphs that could represent the transfer function. Two-flow graph configurations based on Mason's gain formula are of particular interest. To illustrate the derivation of the signal flow graph state model, let us initially consider the fourth order transfer function (7)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{S^4 + a_3 S^3 + a_2 S^2 + a_1 S + a_0} \quad 2.3.5$$

Multiply by s^{-4} then eq 2.3.5 will be

$$G(s) = \frac{b_0 S^{-4}}{1+a_3 S^{-1}+a_2 S^{-2}+a_1 S^{-3}+a_0 S^{-4}} \quad 2.3.$$

First we note that the system is fourth order, and hence we identify four state variables (x_1, x_2, x_3, x_4). Recalling Mason's gain formula, the denominator can be considered to be one minus the sum of loop gains. Furthermore, the numerator of the transfer function is equal to the forward path factor of the flow graph. The flow graph must utilize a minimum number of integrators equal to the order of the system. Therefore we use four integrators to represent this system. The necessary flow graph nodes and the four integrators are shown in fig 2.3.1. Considering the simplest series interconnection of integrators, we can represent the transfer function by the flow graph of fig 2.3.2. Examining this figure, all the loops are touching and that the transfer function of this flow graph is indeed equation (2-3-6) [2]

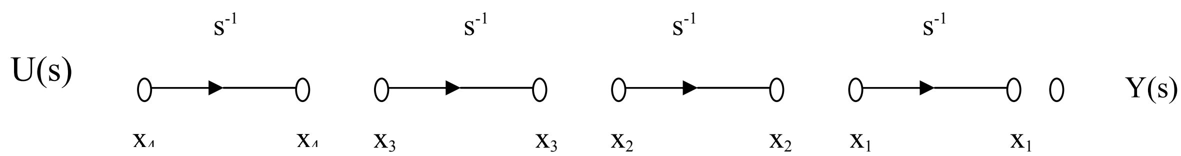


Figure 2-3-1: *Flow graph nodes and integrators for four order system*

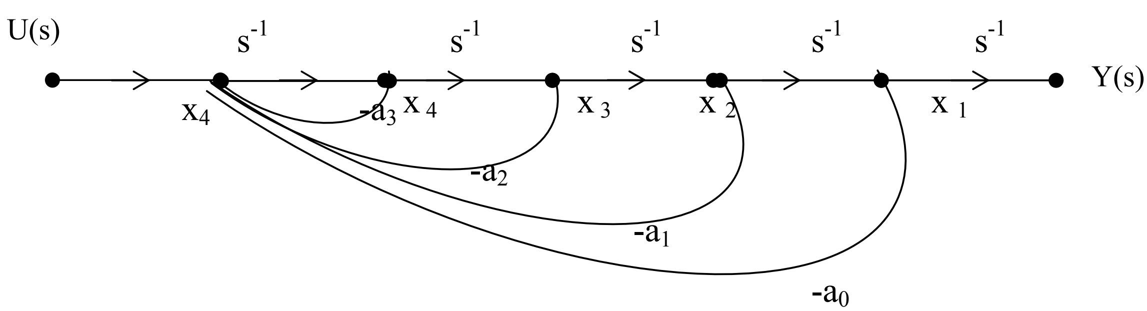


Figure 2.3.2 *Flow Graph State Model G(s) of Eq. 2.3.6*

Consider the fourth order transfer function when the numerator is a polynomial in s so that we have

$$\text{2.} \quad G(s) = \frac{b_3s^3 + b_2s^2 + b_1 s + b_0}{s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

Multiply numerator and denominator by s pow-4

$$\text{2.} \quad G(s) = \frac{b_3s^{-1} + b_2s^{-2} + b_1s^{-3} + b_0 s^{-4}}{1 + a_3s^{-1} + a_2s^{-2} + a_1s^{-3} + a_0s^{-4}}$$

The numerator terms represent forward-path factor in Mason's gain formula. The forward paths will touch all loops, and a suitable signal, flow graph realization of Eq. (2.3.8) is shown in figure (2.3.3). The forward-path factors are b_3/s, b_2/s^2, b_1/s^3, b_0/s^4, as required to provide the numerator of the transfer function. Recall that Mason's flow graph gain formula indicates that the numerator of the transfer function is simply the sum of the forward-path factors. This general form of a signal flow-graph can represent the general transfer function of Eq. (2.3.8) by utilizing n feedback loops involving the (a_n) coefficients and m forward path factors involving the (b_m) coefficients. The general form of the flow graph state model shown in figure (2.3.3) is called the phase variable format.

The state variables are identified in figure (2.3.3) as the output of each energy storage element, that is, the output of each integrator. To obtain the set of first-order of differential equations representing the state model of figure (2.3.3). The signal flow graph, including the added nodes is shown in figure (2.3.4). Using the flow graph of this figure, we are able to obtain the following set of first-order differential equations describing the state of the model.

$$x_1 = x_2, \quad x_2 = x_3, \quad x_3 = x_4 \qquad \text{2.3.9}$$

$$x_4 = -a_0x_1 - a_1x_2 - a_2x_3 - a_3x_4 + u \quad 2.3.10$$

where $x_1, x_2, \ldots\ldots x_n$ are n phase variables.

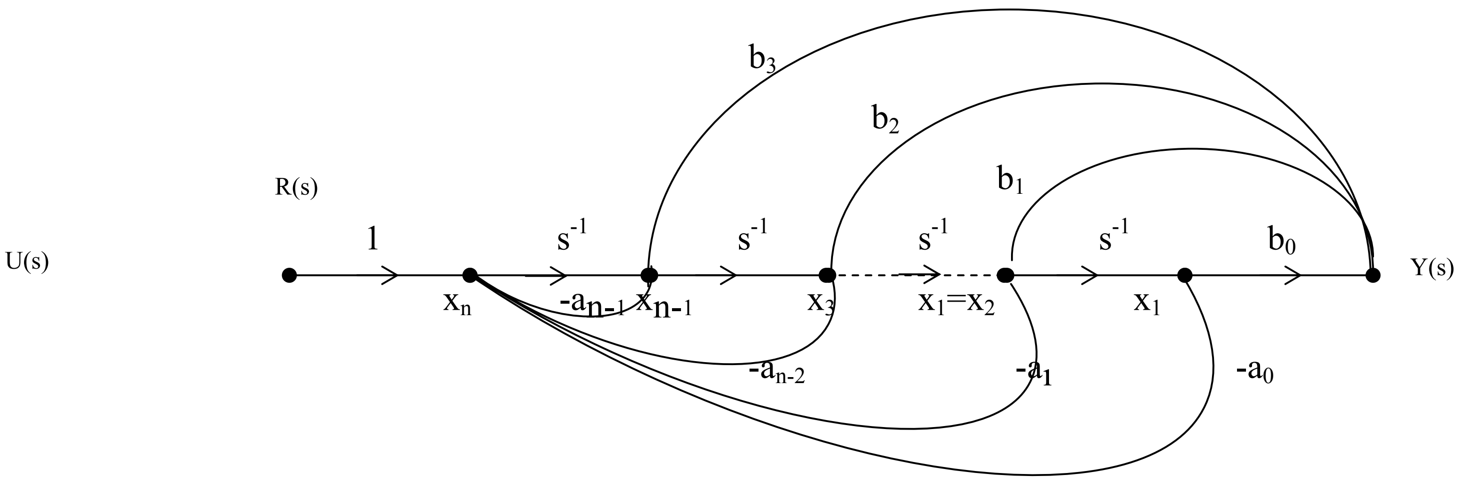


Figure 2.3.3 *Flow-Graph State Model for G(s) of Eq.(2.3.8) in the Phase Variable Format*

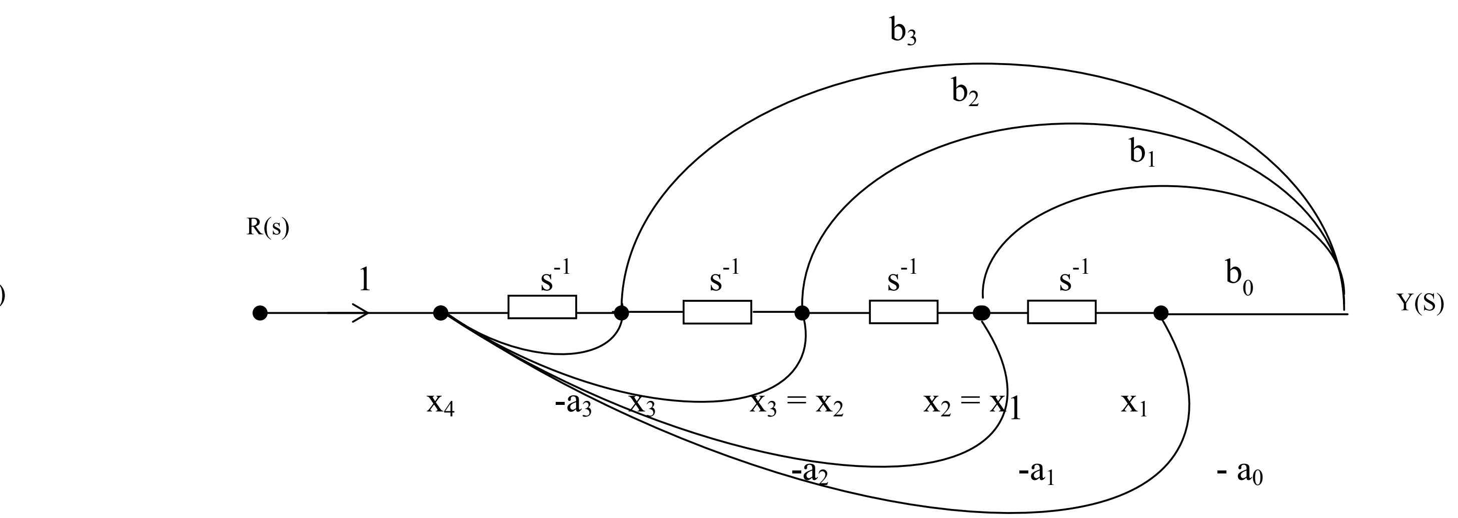


Figure 2.3.4 *Flow Garth of Figure 2.3.3 with nodes inserted*

4- Vector-Matrix Methods

The state variable form of a linear model can be expressed compactly using vector matrix notation. This compactness can be exploited by simulation and analysis techniques. For example, a single computer programs. Similarly, algorithms for determining stability response characteristics can be developed independently of the system order.[4]

4.1- Vector-Matrix Form of the State Equation

The general order of the second–order linear model in term of the state variable x_1, and x_2 is

$$\mathbf{x_1 = a_{11}x_1 + a_{12}x_2 + b_{11}u_1 + b_{12}u_2} \qquad \mathbf{2.4.1}$$

$$\mathbf{x_2 = a_{21}x_1 + a_{22}x_2 + b_{21}u_1 + b_{21}u_2} \qquad \mathbf{2.4.2}$$

Assumed that two inputs (u_1,u_2) act on system, and also two outputs (y_1,y_2). These are linear combination of the state variables and the inputs. Then

$$\mathbf{y_1 = c_{11}x_1 + c_{12}x_2 + d_{11}u_1 + d_{12}u_2} \qquad \mathbf{2.4.3}$$

$$\mathbf{y_2 = c_{21}x_1 + c_{22}x_2 + d_{21}u_1 + d_{21}u_2} \qquad \mathbf{2.4.4}$$

We can from a two dimensional column vector x with two state varieties $x_{1,}x_2$. Similarly for the input vector

$$\mathbf{2.4.5} \qquad x= \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$

$$u = \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} \tag{2.4.6}$$

$$y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix} \tag{2.4.7}$$

From the rules of matrix-vector multiplication, the state of equations and the output equation are be written as:

$$\begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} = \begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} \\ \mathbf{a_{21}} & \mathbf{a_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} + \begin{bmatrix} \mathbf{b_{11}} & \mathbf{b_{12}} \\ \mathbf{b_{21}} & \mathbf{b_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{u_1} \\ \mathbf{u_2} \end{bmatrix} \tag{2.4.9}$$

$$\begin{bmatrix} \mathbf{y_1} \\ \mathbf{y_2} \end{bmatrix} = \begin{bmatrix} \mathbf{c_{11}} & \mathbf{c_{12}} \\ \mathbf{c_{21}} & \mathbf{c_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} + \begin{bmatrix} \mathbf{d_{11}} & \mathbf{d_{12}} \\ \mathbf{d_{21}} & \mathbf{d_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{u_1} \\ \mathbf{u_2} \end{bmatrix}$$

$$\mathbf{x} = \mathbf{Ax} + \mathbf{Bu} \tag{2.4.10}$$

$$\dot{y} = Cy + Du \quad 2.4.11$$

where the matrix A is the system matrix, B is the control or input matrix; C and D are the output matrices. [9].

Chapter III. Design of Controllable/Observable/ Control systems

We first determine an optimal controller for a system described in terms of state variables, and then describe the design of state variable feedback systems with specified characteristic root locations. We also briefly consider the internal model design method and describe the limitations of the state variable feedback methods.[2]

1- The Design of Controllable Control Systems

A system described by the matrices (A, B) can be said to be controllable if there exists an unconstrained control u, that can transfer any initial state x(o) to any other desired location x(t). For system.

$$x = Ax + Bu \quad 3.1.1$$

We can determine whether the system is controllable by examining the algebraic condition rank.

$$[B\ AB\ A^2B\text{-----}A^{n-1}B] \quad 3.1.2$$

For a single-input, single-output system controllability matrix p_c is described in terms of A and b.

$$P_c=[b \quad Ab \quad A^2 \quad \cdot b \quad A^{n-1} \quad b] \tag{3.1.3}$$

which is an nxn matrix. Therefore, if the determinant of p_c , is nonzero, the system is controllable. [2].

Another method of determining whether a system is controllable, is to draw state variable flow graph model and determine whether the control signal, u, has a path to each state variable. If a path to each state exists, then each state variable may be controlled and the system is controllable.

1.1-Controllability Canonical Form CCF

The dynamic equations of a single input single output **SISO**

$$dx(t)/dt = Ax(t) + Bu(t) \tag{3.1.1.1}$$

$$y(t) = Cx(t) + Du(t) \tag{3.1.1.2}$$

where: A, B, C and D are coefficients matrices of appropriate dimensions. The characteristic equation of A is

$$[s1-A] = s^n+a_{n-1} s^{n-1}+ \text{-------} +a_1 s+a_0 = 0 \tag{3.1.1.3}$$

The dynamic equations in Eqs 3.1.1.1 and 3.1.1.2 are transformed into the controllability canonical form (CCF).

$$d\bar{x}(t)/dt = \bar{A}\bar{x}(t) + \bar{B}\bar{u}(t) \tag{3.1.1.4}$$

$$\bar{y}(T) = \bar{C}\bar{x}(t) + D\bar{u}(t) \quad \textbf{3.1.1.5}$$

where $\bar{x}(t)=P^{-1}x(t)$ and p-is an n×n nonsingular matrix

Talking the derivative on both sides of $\bar{x}(t)=P^{-1}x(t)$ with respect to t we have.

$$d\bar{x}(t)/dt = P^{-1}\,dx(t)/dt = P^{-1}Ax(t) + P^{-1}Bu(t) \quad \textbf{3.1.1.6}$$

$$= P^{-1}AP\bar{x}(t) + P^{-1}Bu(t) \quad \textbf{3.1.1.7}$$

Comparing Eg 3.1.1.7 with Eq **3.1.1.4**

$$\bar{A} = P^{-1}AP \quad \textbf{3.1.1.8}$$

$$\bar{B} = P^{-1}B \quad \textbf{3.1.1.9}$$

Using Eq 3.1.1.4, Eq 3.1.1.5 is written

$$\bar{y}(t) = CP\bar{x}(t) + Du(t) \quad \textbf{3.1.1.10}$$

Comparing Eq 3.1.1.10 with 3.1.1.2 we see that

$$\bar{C} = Cp \quad \textbf{3.1.1.11}$$

$$\bar{D} = D \quad \textbf{3.1.1.12}$$

The dynamic equations in Eqs 3.1.1.1 and 3.1.1.2 are transformed into the controllability canonical form (CCF) of the form of Eqs 3.1.1.4 and 3.1.1.1.5 by transformation of Eq 3.1.1.6. With

$$P = SM \quad \textbf{3.1.1.13}$$

where:

$$S = [B \quad AB \quad A^{2-1} \quad B\text{-----------}A^{n-1} \quad B] \qquad \mathbf{3.1.1.14}$$

$$M = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & 1 \\ a_2 & a_3 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 1 & & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \qquad \mathbf{3.1.1.15}$$

$$\bar{A} = P^{-1}AP = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_0 & -a_1 & -a_2 & & -a_{n-1} \end{bmatrix} \qquad \mathbf{3.1.1.16}$$

$$\bar{B} = P^{-1}B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \qquad \mathbf{3.1.1.17}$$

If controllability matrix S is nonsingular, the system can be transformed into the CCF. [1].

1.2- Decomposition of Transfer Functions

It has been shown that the starting point of modeling of a linear system may be the system's differential equation, transfer function, or dynamic equations. It is demonstrated that all these methods are closely related. The block diagram of figure 3.1.2.1 shows the relationships between the various ways of describing a linear system. The block diagram shows that starting, for instance, with the differential equation of a system, one can get to the solution by use of the transfer function method or state equation method. The block diagram also shows that the majority of the relationships are bilateral, so a great deal of flexibility exists between the methods.

One subject remains to be discussed, which involves the constructions of the state diagram from the transfer function between the input and output. The process of going from the T.F. to the state diagram is called decomposition. In general, there are three methods of decomposition of T.F.

Direct decomposition.

Cascade decomposition.

Parallel decomposition {1}.

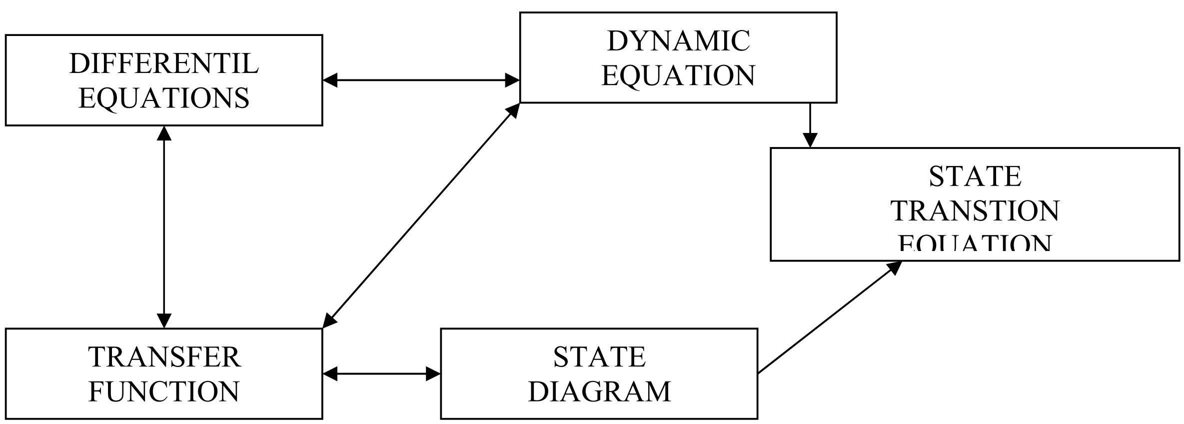


Figure 3.1.2.1 *Block diagram showing the relationships between the methods analysis of a linear system*

1.3-Direct Decomposition

The DD is applied to an input-output T.F that is not in factored form. Consider that the T.F of an nth order SISO system between the input U(s) and output Y(s) is

=

$$\frac{Y(s)}{U(s)} = \frac{b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \text{-----} + b_1 s + b_o}{s^n + a_{n-1} s^{n-1} + \text{------------} + a_1 s + a_0} \qquad \mathbf{3.1.2.1}$$

where the order of the denominator is at least one higher than that of the numerator. The direct decomposition can be conducted in at least two ways, one leading to a state diagram that corresponds to the CCF, the other to the OCF [1]

1.3.1-Direct Decomposition to CCF

To objective is to construct a state diagram from T.F of Eq. 3.1.2.1. The following steps are outlined

1- Express the transfer function in negative powers of s This is done by multiplying the numerator and the denominator of the transfer function by s^{-n}

2-Multiply the numerator and the denominator of the transfer function by a dummy variable X(s). By implementing the last two steps, Eq 3.1.2..1 becomes

$$\frac{Y(s)}{U(s)} = \frac{(b_{n-1} s^{-1} + b_{n-2} s^{-2} + \text{------} + b_1 s^{-n+1} b_o s^{-n})\ X(s)}{(1 + a_{n-1} s^{-1} + a_{n-2} s^{-2} + \text{-----} + a_0 s^{-n})X(s)} \qquad \mathbf{3.1.2.2}$$

3-The numerators and denominators on both sides of Eq 3.1.2.2 are equated to each other, respectively. The results are

$$Y(s) = (b_{n-1} s^{-1} + b_{n-2} s^{-2} + \text{----} + b_1 s^{-n+1} + b_o s^{-n})X(s) \qquad \mathbf{3.1.2.3}$$

$$U(s) = (1 + a_{n-1} s^{-1} + a_{n-2} s^{-2} + \text{-----} + a_0 s^{-n})X(s) \qquad \mathbf{3.1.2.4}$$

4-To construct a state diagram using the two equations in Eqs. 3.1.2.3 and 3.1.2.4, they must first be in the proper cause and effect relation. Equation 3.1.2.4 is rearranged as:

$$\mathbf{X(s) = U(s) \div (a_{n-1}s^{-1} + a_{n-2}\ s^{-2} + --- + a_0 s^{-n})\ X(s)} \qquad \mathbf{3.1.2.5}$$

The state diagram is drawn as shown in fig 3.1.2.2, using Eqs 3.1.2.3 and 3.1.2.5. For simplicity, the initial states are not drawn on the diagram. The state variables x_1 (t), x_2 (t), ----x_n(t) are defined as the outputs of the integrators and arranged in order from the right to the left on the state diagram. The state equations are obtained by applying the SFG gain formula to fig 3.1.2.2, with derivatives of the state variables as the outputs and the state variables and u(t) as the inputs, and overlooking the integrator branches. The output equation is determined by applying the gain formula between the state variables, the input, and the output y (t).

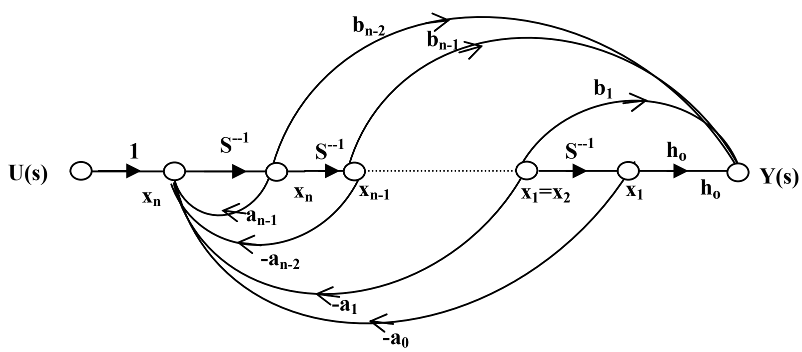


Fig 3.1.2.2 *CCF state diagram of the TF*

The dynamic eqs. are written

$$\mathbf{dx(t)/dt = Ax\ (t) + Bu(t)} \qquad \mathbf{3.1.2.6}$$

$$\mathbf{y(t) = Cx(t) + Du(t)} \qquad \mathbf{3.1.2.7}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \qquad \mathbf{3.1.2.}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{3.1.2.9}$$

$$\mathbf{C} = [\mathbf{b_0} \quad \mathbf{b_1 \ldots\ldots b_{n-2}} \quad \mathbf{b_{n-1}}] \qquad \mathbf{3.1.2.10}$$

$$\mathbf{D} = \mathbf{0} \qquad \mathbf{3.1.2.11}$$

1.4-Controllability of linear Systems

The conditions on controllability and obesrvability essentially govern the existence of a solution to an optimal control problem. This seems to be the basic difference between optimal control theory and classical control theory. In the classical control theory, the design techniques are dominated by trial-and-error methods, so that given a set of design specifications the designer at the outset does not know if any solution exists. Optimal control theory on the other hand, for the most part, has criteria for determining at the outset if the design solution exists or not for the system parameters and design objectives. [10]

The condition of controllability of a system is closely related to the existence of solutions of state feedback for the purpose of placing the eigenvalues of the system arbitrarily. The concept of observability relates to the condition of observing or estimating the state variables from the output variables, which are generally measurable.

One way of illustrating the motivation of investigating controllability and observability can be made by referring to the block diagram shown in figure 3.1.2.1. Figure 3.1.2.1 shows a system with the process dynamics described by:

$$dx(t)/dt = Ax(t) + Bu(t) \quad 3.1.3.1$$

Feeding back the state variables through a constant feedback gain matrix [K] forms the closed loop system

$$u(t) = -Kx(t) + r(t) \quad 3.1.3.2$$

where K is a pxn feedback matrix with constant elements. The closed loop system is thus described by

$$dx(t)/dt = (A - BK)\, x(t) + Br(t) \quad 3.1.3.3$$

This problem is also known as the pole-placement design through state feedback. The design objective in this case is to find the feedback matrix K such that the eigenvaluses of (A-BK), or of the closed-loop system, are of certain prescribed values. The result is that if the system of eq 3.1.3.1 is controllable, there exists a constant-feedback matrix K that allows the eigenvalues of (A-BK) to be assigned arbitrarily.[4]

Once the closed-loop system is designed, one has to deal with the practical problem of implementing the feedback of state variables. There are two practical problems with implementing state feedback control. One is that the number of state variables may be excessive, so that cost of sensing each of these state variables for feedback may be prohibitive. The other problem is that not all the state variables are physically accessible. Therefore,. it may be necessary to design and construct an observer that will estimate the state vector from the output vector y(t). [9]

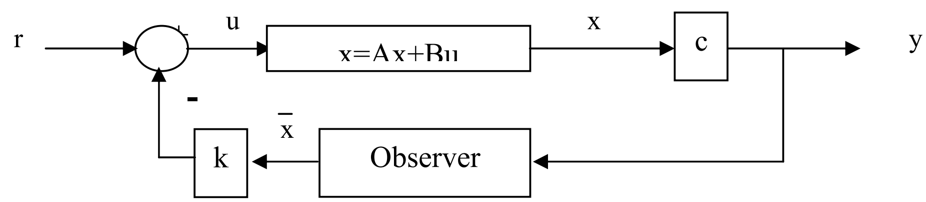


Figure 3-1-2-1 *control system with observer and state feedback*

The observed state vector x (t) is used to generate the control u (t) through the feedback matrix K. The condition that such an observer can be designed for the system is called the observability of the system.[1].

1.5-General concept of controllability:

The concept of controllability can be stated with reference to the block diagram of figure 3.1.2.2

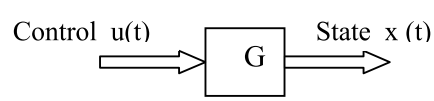


Fig 3.1.2.2 *linear time invariant system*

The process is said to be completely controllable if every state variable of the process can be controlled to reach a certain objective in finite time by some unconstrained control u(t). Intuitively, it is simple to understand that if any one of the state variables is independent of the control u(t), there would be no way of driving this particular state variable to a desired state in finite time by means of a control effort [1].

1.6- Definition of state controllability

Consider that a linear time-invariant system is described by the following dynamic equations.

$$\mathbf{dx(t)/dt = Ax(t) + Bu(t)} \qquad \mathbf{3.1.5.1}$$

$$\mathbf{y(t) = Cx(t) + Du(t)} \qquad \mathbf{3.1.5.2}$$

where x(t) is the nx1 state vector, u(t) is rx1 input vector, and y(t) px1 is output vector A,B,C and D are coefficients of appropriate dimensions.

The state x(t) is said ~~to~~ be controllable at t= t_0 if there exists a piecewise-continuous input u(t) that will drive the state to any final state $x(t_f)$ for a finite time $(t_f-t_0) \geq 0$. [1].

Theorem 1

For the system described by the state equation of eq (3.1.5.1) to be completely state controllable, it is necessary and sufficient that the following n * n controllability matrix has a rank of n.

$$\mathbf{S = [B \quad AB \quad A^2 \quad B \quad \ldots\ldots A^{n-1} \quad B]} \qquad \mathbf{3.1.5.3}$$

Since the matrices A and B are involved, sometimes we say that the pair [A,B] is controllable which implies that S is of rank n

1.7- Alternative tests on controllability

There are several other alternative methods of testing controllability, and some of these may be more convenient to apply than the condition in eq 3.1.5.3

Theorem (2)

For a SISO system described by the state equation of eq (3.1.5.1),the pair [A,B] is completely controllable if A and B are in CCF or transformable into CCF by similarity transformation.

The proof of this theorem is straightforward, since it was established, in that the CCF transformation requires that the controllability matrix S be nonsingular. Theorem applies only for a SISO system. [1]

Theorem [3]

For a system described by the state equation of Eq 3.1.5.1 if A is in DCF or JCF, the pair [A,B] is completely controllable if all the elements in the rows of B that correspond to the last row of each Jordan block are nonzero.

The proof of this theorem comes directly from the definition of controllability. Let us assume that A is diagonal and that it has distinct eigenvalues. Then the pair [A, B] is controllable if B does not have any row with all zeros. The reason is that if A is diagonal, all the states are decoupled from each other, and if any row of B contains all zero elements, the corresponding state would not be accessed from any of the inputs, and that state would be uncontrollable.

For a system in JCF, such as the A and B matrices illustrated below, for controllability, only the elements in the row of B that corresponds to the last row of the Jordan block are not all zeros. The elements in the other rows of B need not all be nonzero, since the corresponding states are still coupled through the first in the Jordan blocks of A.

$$\left[\begin{array}{ccc|c} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ \hline 0 & 0 & 0 & \lambda_2 \end{array}\right]$$

A = ------

3.1.6.1

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}$$

3.1.6.2

Thus the condition of controllability for the A and B in Eq 3.1.6.2, is $b_{32} \neq 0$, $b_{31} \neq 0$, $b_{41} \neq 0$, $b_{42} \neq 0$ [1].

1.8. Invariant theorem on controllability:

The effects of controllability due to state feedback will be investigated [6]

Theorem (4) Invariant theorem on similarity transformation

Consider that the system described by dynamic equations of Eqs 3.1.1 and 3.1.2. The similarity transformation $x(t) \bar{=} P\ x(t)$, where P is nonsingular, transforms the dynamic equations to

$$\mathbf{d\bar{x}(t)/d(t) = \bar{A}\ x(t) + \bar{B}\ u(t)}$$

3.1.7.1

$$\mathbf{\bar{y}(t) = \bar{C}\ x(t) + Du(t)}$$

3.1.7.2

where $\mathbf{\bar{A} = P^{-1}AP}$; $\mathbf{\bar{B} = P^{-1}B}$

The controllability of $[\bar{A}, \bar{B}]$ is not affected by the transformation. Showing the rank of S easily proves the theorem $\bar{}$and S are identical, where S is controllability matrix of the transformed system.

Theorem 5 theorem on controllability of closed-loop system with state feedback

If an open-loop system

$$dx(t)/dt = Ax(t) + Bu(t) \quad 3.1.7.3$$

is completely controllable, the closed loop system obtained through state feedback.

$$u(t) = r(t) - k\, x(t) \quad 3.1.7.4$$

Substituting Eq 3.1.7.4 into Eq 3.1.7.3 state equation becomes

$$dx(t)/dt = (A - Bk)x(t) + Br(t) \quad 3.1.7.5$$

is also completely controllable. On the other hand if [A, B] is uncontrollable, there is no K that will make the pair [A-BK,B] controllable. [1]

2.Design of Observable Control Systems

All the roots of the characteristic equation can be placed where desired in the s-plane if, and only if, a system is observable and controllable. Observability refers to the ability to estimate a state variable. The system is observable if the output has component due to each state variables. Another way of stating the requirement is to prescribe that a path exist between each state variables and the output variable on the system flow-graph model

A system is observable if, and only if, there exist a finite time T such that the initial state x(0) can be determined from the observation history y(t) given the control u(t).

Consider the single-input, single output system.

$$\mathbf{x} = \mathbf{Ax} + \mathbf{bu} \tag{3.2.1}$$

$$\mathbf{y} = \mathbf{dx} \tag{3.2.2}$$

where d is a row vector and x is a column vector. This system is observable when the determinant of Q is nonzero where

$$\mathbf{Q} = \begin{bmatrix} d \\ dA \\ \vdots \\ dA^{n-1} \end{bmatrix} \tag{3.2.3}$$

Alternatively, we can draw the flow graph model of system and determine whether every state variable has a path to the output y(t). A system that can be described in the phase variable format is always observable. [2]

2.1 Observability Canonical form (OCF)

A dual form of transformation of the CCF is the observability canonical form OCF. The system described by eqs 3.2.1 and 3.2.2 is transformed to the OCF by the transformation

$$\bar{x}(t) = Qx(t) \qquad 3.2.1.1$$

The transformed equations are given in Eqs 3.1.1.4 and3.1.1.5

$$\bar{A} = Q^{-1} AQ \qquad 3.2.1.2$$

$$\bar{B} = Q^{-1}B \qquad 3.2.1.3$$

$$\bar{C} = CQ \qquad 3.2.1.4$$

$$\bar{D} = D \qquad 3.2.1.5$$

where

$$\bar{A}=Q^{-1}AQ = \begin{bmatrix} 0 & 0 & 0\text{----------}0 & -a_0 \\ 1 & 0 & 0\text{----------}0 & -a_1 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & 0\cdots\cdots 1 & -a_{n-} \end{bmatrix} \qquad 3.2.1.6$$

$$\bar{C}= CQ=[0 \quad 0 \quad \ldots\ldots\ldots\ldots 0 \quad 1] \qquad 3.2.1.7$$

The elements of the matrices $\bar{B}$ and $\bar{D}$ are not restricted to any form. Notice that $\bar{A}$ and $\bar{C}$ are the transpose of the A and B respectively. The OCF transformation matrix Q is given by

$$Q = (MV)^{-1} \qquad 3.2.1.8$$

where M is a given in Eq 3.1.1.15 and

$$V=\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad (n*n)$$

3.2.1.9

The matrix V is often defined as the observability matrix V^{-1}, and must exist in order for the OCF transformation to be possible. [1]

2.2- Direct Decomposition to OCF

Multiplying the numerator and the denominator of Eq3.1.2.1 by s^{-n} the equation is expended as

$$\mathbf{(1 + a_{n-1}\, s^{-1} + --- a_1\, s^{-n+1} + a_o\, s^{-n})\, Y(s) = (b_{n-1}\, s^{-1} + b_{n-2}\, s^{-2} + --- + b_1 s^{-n+1} + b_o s^{-n})\, U(s)}$$

3.2.2.1

or

$$\mathbf{Y(s) = -\,(a_{n-1}s^{-1} + --- + a_1\, s^{-n+1} + a_o s^{-n})\, Y(s) + (b_{n-1}s^{-1} + b_{n-2}s^{-2} + --- + b_1 s^{-n+1} + b_o s^{-n})\, U(s)}$$

3.2.2.2

The state diagram form will be following

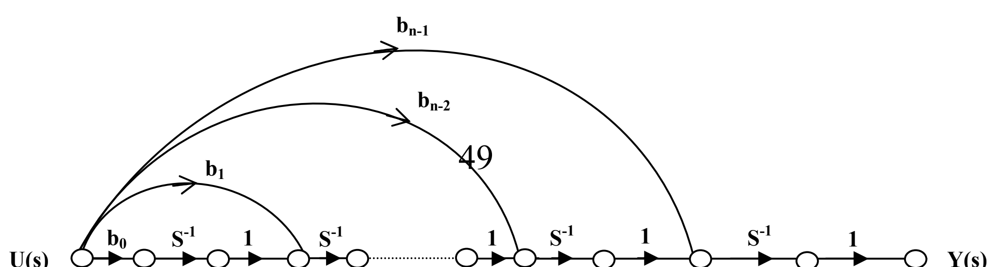


Fig. 3.2.2.1: *OCF state diagram of the T.F in Eq 3.2.2.1*

The outputs of the integrators are designated as the state variables. The dynamic equations are written as in eqs 3.1.1.1 and 3.1.1.2, with

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ 0 & 1 & \cdots & 0 & -a_2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-2} \\ b_{n-1} \end{bmatrix} \qquad \mathbf{3.2.2.3}$$

C = [0 0 .... 0 1] **D = 0**

3.2.2.3

The matrices A and C are of OCF.

2.3- Observabiliy of Linear Systems

Essentially, a system is completely observable if every state variable of the system affects some of the outputs. If any one of the states cannot be observed from the measurements of the outputs, the state is said to be unobservable, and the system is not completely observable, or simply unobservable. Figure (3.2.3.1) shows the state diagram of a linear system in which the state x_2 is not connected to the output y(t) in any way. Once we have measured y(t), we can observe the state x_1 (t), since $x_1(t) = y$ (t). However, the state x_2 cannot be observed from the information on y(t). Thus the system is unobservable [1].

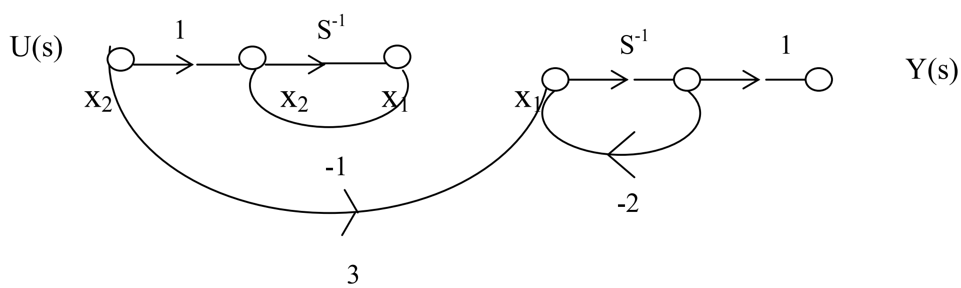


Figure 3.3.1 *State diagram of a system that is not observable.*

2.4- Definition of Observability

Given a linear time-invariant system that is described by the dynamic equations of Eqs (3.1.5.1) and 3.1.5.2 the state $x(t_0)$ is said to be observable if given any input u(t), there exits a finite time $t_f \geq t_0$ such that the knowledge of u(t) for $t_0 \leq t \leq t_f$, matrices A,B,C and D, and the output y(t) for $t_0 \leq t \leq t_f$ are sufficient to determine $x(t_0)$. If every state of the system is observable for a finite t_f, we say that the system is completely observable or simply observable. The condition of observability depends on the matrices A and C of the system. The following theorem shows that

Theorem [6]

For the system described by Eqs (3.1.5.1) and (3.1.5.2) to be completely observable, it is necessary and sufficient that the following n* n observability matrix has a rank of n

$$\mathbf{V} \quad \mathbf{3.2.2.4} \quad \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad =$$

The system is completely observable if V is nonsingular. [2]

2.5-Alternative Test on Observability

These are several alternative methods of testing observability

Theorem [7]

For a SISO system described by the dynamic equations of Eqs 3.1.5.1 and 3.1.5.2, the pair {A,C} is completely observable if A and C are in OCF or transformable into OCF by similarity transformation.

Theorem [8]

The a system described by the dynamic equations Eqs (3.1.5.1) and (3.1.5.2) if A is in DCF or JCF the pair {A, C} is completely observable if all the elements in the columns of C that correspond to the first row of each Jordan block are nonzero. [1]

2.6- Invariant theorem on observability:

The effects of observability due to state feedback will be investigated. See theorem 4 and consider that the system described by the dynamic equations Eqs (3.1.5.1) and (3.1.5.2)

The observability of $(\bar{A}, \bar{C})$ is not affected by transformation. In other words observability is preserved through similar transformations. The theorem is easily proven by showing that the rank of V and $\bar{V}$ are identical, where $\bar{V}$ is observability matrix, of the transformed system.

Theorem (9)

If an open loop system is observable, the state feedback of the form of Eq 3.1.1.7 could destroy observability. In other wards, the observability of open loop and closed-loop systems due to state feedback are unrelated.

Chapter IV: Computer Software for controllable And Observable Control Systems

In recent years, the analysis of control systems has been affected dramatically by widespread use of computers, especially personal computers. Personal computers have become so powerful and advanced that they can be used to solve sophisticated and complex control systems problems with ease.

The following software is all available in both PC and Macintosh versions.

1. ACSP Version 2.0 by Benjamin C.Kuo.

2. MATLAB Tools for control system analysis and Design Second Edition by D.C.

3. CSAD is a menu - and mouse driver set of MATLAB Tools to supplement undergraduate studies of continuous - data control systems. The following prescriptions show what the tools are and are not, and the important M-files.

- IT IS a set of (over 70) M-files that will run on any 3.5 version or 5.3.1 version of MATLAB. The control systems Toolbox and signals and systems toolbox and not needed.

In this chapter we used MALAB version 5.3.1 tools for solving of analysis of controllable and observable control systems. Practically we use high-order model system matrix with different value:

det (A) = 0

det (A) = 225

det (A) = -209

Also the control systems matrix (B) by different column matrix, and choose which control system is better for controllability canonical form. Also use proceeding system matrix, which control system better for (OCF), and method how we can improve the CCF and OCF.

In this chapter also we solve the feedback analysis and show, what the affect of feedback of design of controllable and observable control systems. In this part we used the third-order differential equation.

In the last of this chapter we proved the result of proceeding analysis in Magnetic-ball. Suspension. And also in laboratory using software, we did four experiments:

Table 4.1*: Controllability and Uncontrollability condition of Control System when det(A)=0*

det (A)	B	Det(S)	Result	Condition
0	[1;0;0;0;0]	215689	Non Singular	Controllable
	[0,1;0;0;0]	-696	Non Singular	Controllable
	[0;0;1;00]	0	Singular	Uncontrollable
	[0;0;0;1;0]	0	Singular	Uncontrollable
	[0;0;0;0;1]	0	Singular	Uncontrollable
	[1;1;0;0;0]	0	Non Singular	Uncontrollable
	[1;0;1;0;0]	100764	Non Singular	Controllable
	[1;0;0;1;0]	45756	Non Singular	Controllable
	[1;0;0;0;1]	-60756	Non Singular	Controllable
	[0;1;1;0;0]	-100620	Non Singular	Controllable
	[0;1;0;1;0]	-396	Non Singular	Controllable
	[0;1;0;0;1]	142500	Non Singular	Controllable
	[0;0;1;1;0]	0	Singular	Uncontrollable
	[0;0;1;0;1]	0	Singular	Uncontrollable
	[0;0;0; 1;1]	0	Singular	Uncontrollable
	[1;1;1;0;0]	0	Singular	Uncontrollable
	[1;1;0;1;0]	0	Singular	Uncontrollable

	[1;1;0;0;1]	0	Singular	Uncontrollable
	[0;1;1;1;0]	-81120	Non Singular	Controllable
	[0;1;0;1;1]	105000	Non Singular	Controllable
	[0;1;1;0;1]	9968	Non Singular	Controllable
	[0;0;1;1;1]	0	Singular	Uncontrollable
	[1;1;1;1;0]	0	Singular	Uncontrollable
	[1;1 ;1 ;0;1]	0	Singular	Uncontrollable
	[1;1;0; 1;1]	0	Singular	Uncontrollable
	[1;0;1;1;1]	22500	Non Singular	Controllable
	[0;1;1;1;1]	8460	Non Singular	Controllable
	[1;1.1.1.1]	0	Singular	Uncontrollable

Table 4.2: *Controllability and Uncontrollability condition of Control System when det(A)=225*

Det (A)	B	Det(S)	Result	Condition
	[10;0;0;0]	215689	Non Singular	Controllable
	[0;1;0;0;0]	71492	Non Singular	Controllable
	[0;0;1;0;0]	12177	Non Singular	Controllable
	[0;0;0;1;0]	-68308	Non Singular	Controllable
	[0;0;0;0;1]	-28800	Non Singular	Controllable
	[1;1;0;0;0]	9512175	Non Singular	Controllable
	[1;0;1;0;0]	49700	Non Singular	Controllable
	[1;0;0;1;0]	967401	Non Singular	Controllable
	[1;0;0;0;1]	4713641	Non Singular	Controllable
	[0;1;1;0;0]	5785839	Non Singular	Controllable
	[0;1;0;1;0]	-126320	Non Singular	Controllable
	[0;1;0;0;1]	55380	Non Singular	Controllable
	[0;0;1;1;0]	13541955	Non Singular	Controllable
	[0;0;1; 0;1]	3120255	Non Singular	Controllable
	[0;0;0;1;1]	540	Non Singular	Controllable
225	[1;1;1;0;0]	48640	Non Singular	Controllable
	[1;1;0;1;0]	8431605	Non Singular	Controllable
	[[1;1;0;0;1]	4792825	Non Singular	Controllable
	[1;0;1;1;0]	7679440	Non Singular	Controllable
	[1;0;1;0;1]	404100	Non Singular	Controllable

	[1;0;0;1;1]	4121827	Non Singular	Controllable
	[0;1;1;1;0]	29112119	Non Singular	Controllable
	[0;1;1;0;1]	2466163	Non Singular	Controllable
	[0;1;0;1;1]	-2434240	Non Singular	Controllable
	[0;0;1;1;1]	32613055	Non Singular	Controllable
	[1;1;1;;1;0]	835340	Non Singular	Controllable
	[1;1;1;0;1]	-451072	Non Singular	Controllable
	[1;1;0;1;1]	1266005	Non Singular	Controllable
	[1;0;1;1;1]	12415424	Non Singular	Controllable
	[0;1;1;1;1]	3121193	Non Singular	Controllable

Table 4.3: *Controllability and Uncontrollability condition of Control System when det(A)=-209*

Det (A)	B	Det(S)	Result	Condition
- 209	[1;0; 0;0;0]	-775853	Non singular	Controllable
	[0;1;0;0;0]	-20302	Non singular	Controllable
	[0;0;1;0;0]	753099	Non singular	Controllable
	[0;0;0;1;0]	3670	Non singular	Controllable
	[0;0;0;0;1]	562692	Non singular	Controllable
	[1;1;0;0;0]	-5725821	Non singular	Controllable
	[1;0;1;0;0]	-112034	Non singular	Controllable
	[1;0;0;1;0]	-3506393	Non singular	Controllable
	[1;0;0;0;1]	-1449149	Non singular	Controllable
	[0;1;1;0;0]	82937	Non singular	Controllable
	[0;1;0;1;0]	28864	Non singular	Controllable
	[0;1;0;0;1]	176426	Non Singular	Controllable
	[0;0;1;1;0]	177417	Non Singular	Controllable
	[0;0;1;0;1]	14311207	Non Singular	Controllable
	[0;0;0;1;1]	177522	Non Singular	Controllable
	[1;1;1;0;0]	-26160	Non Singular	Controllable
	[1;1;0;1;0]	14279337	Non Singular	Controllable
	[1;1;0;0;1]	-1893205	Non Singular	Controllable
	[0;1;1;1;0]	551	Non Singular	Controllable
	[1;0;1;1;0]	-5828	Non Singular	Controllable
	[1;0;0;1;1]	160376	Non Singular	Controllable
	[0;0;1;1;1]	4248723	Non Singular	Controllable
	[1;1;1;1;0]	-79950	Non Singular	Controllable

	[0;1;1;1;1]	1300912	Non Singular	Controllable
	[1;0;1;1;1]	-595640	Non Singular	Controllable
	[1;0;1;1;1]	-595640	Non singular	Controllable
	[1;1;0;1;1]	-3194475	Non singular	Controllable
	[1;1;1;0;1]	-1943868	Non Singular	Controllable
	[1;1;1;1;0]	-79950	Non Singular	Controllable
	[1;;1;1;1;1]	-630306	Non Singular	Controllable

The design of controllable control systems described by the matrices [A, B]. We can determine whether the system is controllable by examining the algorithmic condition rank

S= [B AB A^2B---- A^{n-1}B]

which is S an n * n matrix, therefore if the determinant of S is nonzero, the system is controllable.

Practically we determine the condition of system by different of A matrix. In table 4.1 shown det (A) = 0, table 4.2 det (A) = 225, and table 4.3 det (t) = -209. If the determinant of matrix (A) is positive or negative, the control systems are fully controllable by any input-signal connection (tables 4.2, 4.3). We can be improved by adding another input-signal for controllability. When the determinant of system matrix is zero (table 3.1), the systems are uncontrollable in many input-signals connections. This situation can be changed to controllability by adding another input-signal.

Table 4.4: *Observability and Un-observability condition of Control System* when det(A)=0

Det (A)	B	Det(V)	Result	Condition
0	[10000]	0	Singular	Unobservable
	[01000]	0	Singular	Unobservable
	[00100]	0	Non Singular	Unobservable
	[00010]	429960	Singular	Observable
	[00001]	0	Singular	Unobservable
	[11000]	0	Singular	Unobservable
	[10100]	0	Singular	Unobservable
	[10010]	193856	Non Singular	Observable
	[10001]	0	Singular	Unobservable
	[01100]	0	Singular	Unobservable
	[01001]	-0	Singular	Unobservable
	[00110]	106760	Non Singular	Observable
	[00101]	0	Singular	Unobservable
	[00011]	2051072	Non Singular	Observable
	[11100]	0	Singular	Unobservable
	[11010]	-43168	Non Singular	Observable
	[11001]	0	Singular	Unobservable
	[10110]	-312000	Non Singular	Unobservable
	[10101]	-0	Singular	Unobservable
	[10011]	1538096	Non Singular	Observable
	[01110]	-352600	Non Singular	Observable
	[01011]	1757824	Non Singular	Observable
	[01101]	0	Singular	Unobservable
	[00111]	2275200	Non Singular	Observable
	[11110]	-749400	Non Singular	Observable
	[11101]	0	Singular	Unobservable
	[11011]	1145984	Non Singular	Observable
	[10111]	1515904	Non Singular	Observable
	[01111]	1705392	Non Singular	Observable
	[11111]	8929928	Non Singular	Observable

Table 4.5: *Observability and Un-observability condition of Control System when det(A)=225*

Det (A)	C	Det (V)	Result	Condition
225	[10000]	373531	Non singular	Observable
	[01000]	438741	Non singular	Observable
	[00100]	-31197	Non singular	Observable
	[00010]	-293940	Non singular	Observable
	[00001]	1424228	Non singular	Observable
	[11000]	-137916	Non singular	Observable
	[10100]	164448	Non singular	Observable
	[10010]	27624981	Non singular	Observable
	[10001]	1464237	Non singular	Observable
	[01100]	3274764	Non singular	Observable
	[01010]	6527	Non singular	Observable
	[01001]	3289553	Non singular	Observable
	[00101]	1124513	Non singular	Observable
	[00110]	7918925	Non singular	Observable
	[00011]	1308496	Non singular	Observable
	[11100]	-911505	Non singular	Observable
	[11010]	7776000	Non singular	Observable
	[11001]	931152	Non singular	Observable
	[10110]	2024964	Non singular	Observable
	[10011]	-105759	Non singular	Observable
	[01110]	-259840	Non singular	Observable
	[01101]	2455312	Non singular	Observable
	[01011]	4299385	Non singular	Observable
	[00111]	33321185	Non singular	Observable
	[11110]	9032805	Non singular	Observable
	[11101]	8671275	Non singular	Observable
	[11011]	7812596	Non singular	Observable
	[10111]	535040	Non singular	Observable
	[01111]	6272316	Non singular	Observable
	[11111]	8950883	Non singular	Observable

Table 4.6: *Observability and Un-observability condition of Control System when det(A)=-209*

Det (A)	C	Det (V)	Result	Condition
	[10000]	-14597	Non singular	Observable
	[01000]	131905	Non singular	Observable
	[00100]	-443769	Non singular	Observable
	[00010]	-1099186	Non singular	Observable
	[00001]	8138	Non singular	Observable
	[11000]	-1486174	Non singular	Observable
	[10100]	--347404	Non singular	Observable
	[10010]	-10782519	Non singular	Observable
	[10001]	-114867	Non singular	Observable
	[01100]	-5425142	Non singular	Observable
	[01010]	-2696847	Non singular	Observable
	[01001]	594261	Non singular	Observable
	[00101]	-527305	Non singular	Observable
	[00110]	-29214145	Non singular	Observable
	[00011]	-24284	Non singular	Observable
	[11100]	-22724719	Non singular	Observable
- 209	[11001]	-1423988	Non singular	Observable
	[10110]	-91814034	Non singular	Observable
	[10101]	-4696386	Non singular	Observable
	[10011]	-11084155	Non singular	Observable
	[01110]	-64708680	Non singular	Observable
	[01101]	-4388460	Non singular	Observable
	[01011]	2278931	Non singular	Observable
	[00111]	-24473303	Non singular	Observable
	[11110]	-188273609	Non singular	Observable
	[11101]	-26039177	Non singular	Observable
	[11011]	-26115990	Non singular	Observable
	[10111]	-90449164	Non singular	Observable
	[01111]	-52084174	Non singular	Observable
	[11111]	-183803493	Non singular	Observable

The matrices [A, C] describe the design of observable control systems. We can determine whether the system is observable by examining the algebraic condition rank.

$$V = \begin{bmatrix} C \\ CA \\ - \\ - \\ C*A^{n-1} \end{bmatrix}$$

which is (V) an n*n matrix, therefore, if the determinant of (V), is nonzero, the control system is observable.

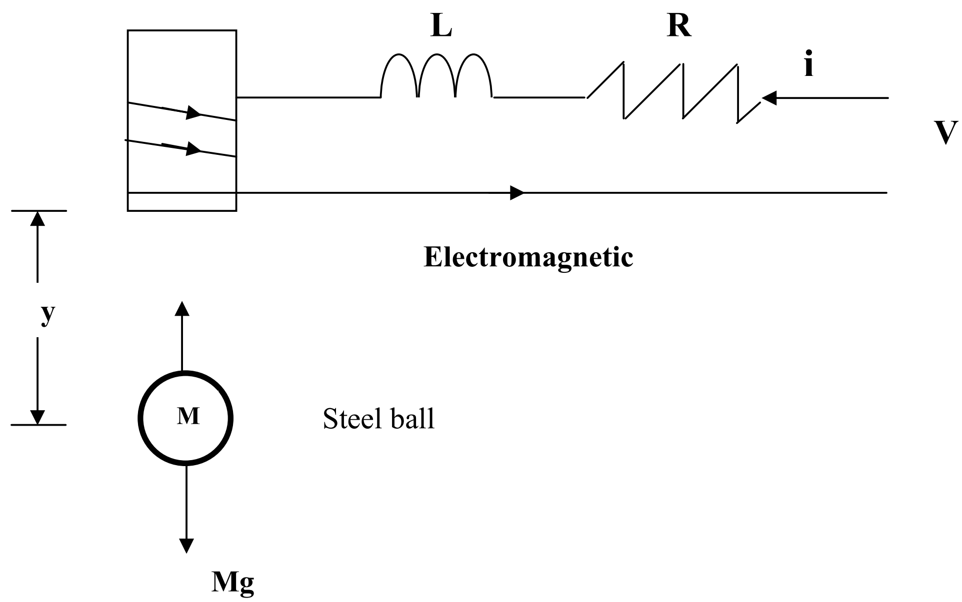
Practically we determine the condition of the system by different value of A matrix. In table 4.4 det(A) = 0, table 4.5 det (A) =225, and table 4.6 det (A) =-209. If the determinant of system matrix (A) is positive or negative, the control systems are fully observable by any output connection tables (4.5, 4.6). We can be improved observability by adding another output. When the determinant of system matrix is zero (table 4.4) the control systems are unobservable in many outputs connections. To change this situation to observability can be done by adding another output.

7. Magnetic Ball Suspension System

Consider the magnetic ball suspension system shown in fig. (4.1). The objective of the system is to regulate the current of electromagnet so that the ball will be suspended at a fixed distance from the end of the magnet. The dynamic equations of the system are

$$M \frac{d^2 x(t)}{dt^2} = Mg - \frac{Ki^2(t)}{x(t)}$$

$$V(t) = R(t) + L \frac{d\,i(t)}{dt}$$



The system variables and parameters are as follows:

V(t) – input voltage (V). K – proportion constant

I(t) - winding current (A). L – winding inductance = 0.02 H

R - winding resistance = 4 Ω. M – ball mass = 0.5 Kg

X(t) – ball positoin (m). g – gravitation acceleration = 9.8 m/ sec

The state variables are defined as

$$\mathbf{x_1(t) = x(t)}$$

$$\mathbf{x_2(t) = dx(t) / dt}$$

$$\mathbf{x_3(t) = i(t)}$$

The state eguations are

$$\mathbf{dx_1(t) / dt = x_2 (t)}$$

$$\frac{\mathbf{dx_2 (t)}}{\mathbf{dt}} = \mathbf{g} - \frac{\mathbf{k\, x_3{}^2(t)}}{\mathbf{M\, x_1 (t)}}$$

$$\frac{\mathbf{x_3(t)}}{\mathbf{dt}} = - \frac{\mathbf{R}}{\mathbf{L}} \mathbf{x_3(t)} + \frac{\mathbf{v(t)}}{\mathbf{L}}$$

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

```
» global v; % input voltage (V):
» global i; % winding current(A):
» global R; % Winding resistance (Omm):
» global g; % gravitational acceleration (m/Sec^2)
» global M; % Ball mass (m):
» global x; % Ball position (m):
```

```
» global K; % Proportional constant:
» global L; % winding inductance (H):
» R=4
R =
 4
» g=9.8
g =
   9.8000
» M=0.5
M =
   0.5000
» x=0.5
x =
   0.5000
» K=1.00
K =
     1
» L=0.02
L =
   0.0200
» x1=x
x1 =
   0.5000

» x2=diff(x);
» x3=i
x3 =
     []
» x2=diff(x1);
» diff(x2)=g-(K/M)*(x3)^2;
» diff(x3)=-(R/L)*x3-(1/L)*v;
» diff(x2)=9.8*x1-16*x3;
» diff(x3)=-200*x3-50*v;
» A=[0 1 0;9.8 0 16;0 0 -200], % System matrix:
```

```
A =
        0          1.0000      0
        9.8000     0           16.0000
        0          0           -200.0000
» B=[0;0;1], % Control matrix:
B =
    0
    0
    1
```

Use single quote character instead of double quote or backward quote.

```
» G=sym('[s 0 0;0 s 0;0 0 s]')
G =
[ s, 0, 0]
[ 0, s, 0]
[ 0, 0, s]
» G-A
 ans =
[  s,         -1,     0]
[ -49/5,      s,   -16]
[     0,     0, s+200]

» det(G-A)
 ans =
s^3+200*s^2-49/5*s-1960
» P=[1 200 9.8 -1960]
P =
  1.0e+003 *
    0.0010    0.2000    0.0098   -1.9600
» r=roots(P)
r =
 -199.9019
   -3.1807
    3.0826
» S=[B A*B A^2*B]
```

```
S =
        0         0        16
        0        16     -3200
        1      -200     40000
» det(S)
ans =
-256
» C=[1 0 0], % Ball position:
C =
   1    0    0
» V=[C;C*A;C*A^2]
V =
      1.0000      0          0
      0           1.0000     0
      9.8000      0          16.0000
» det(V)
ans =
   16
» C=[0 1 0], % Ball velocity:
C =
   0    1    0
» V=[C;C*A;C*A^2]
V =
1.0e+003 *
   0          0.0010       0
   0. 0098    0            0.0160
   0          0.0098      -3.2000
» det(V)
ans =
  3.1360e+004
» C=[0 0 1], % Winding current:
C =
   0    0    1
» V=[C;C*A;C*A^2]
```

V =

0	0	1
0	0	-200
0	0	40000

» det(V)

ans =

0

Chapter V: Examples of SISO Systems

1- Open-Loop Control of a DC Motor

1.1- Objectives

* To control the speed of a dc motor at no-load.

* To measure the steady-state open-loop gain of a control system.

* To observe the dead-bond of a control system.

When we have completed this example we will know what is meant by open- loop gain and how it can be changed. We will also discover that amplifier saturation has a serious effect on system gain.

1.2- Equipment Required

Description

Model

Enclosure / Power Supply

8846

Connections Leads and Accessories

8944

Potentiometer

9036

Power Amplifier

9039

Dc Motor / Generator

9318

1.3- Discussion of Fundamentals

Consider the simple control system shown in figure 5-1.1, to vary the speed of a dc motor. It consist of a potentiometer P, a power amplifier A, a dc motor M, and a tachometer G. The motor is geared down to drive an output shaft at low speed. The potentiometer connected to the reference supply enables us to vary voltage V_1. This is the input signal to power amplifier A. A change in V_1 will produce a change in the armature voltage V_a. Our knowledge of dc motors tells us that the speed increases with an increase in armature voltage. Furthermore, the dc tachometer acts as a transducer because it gives a voltage V_2 that is directly proportional to speed. Consequently, an increase in V_1 produces an increase in V_2

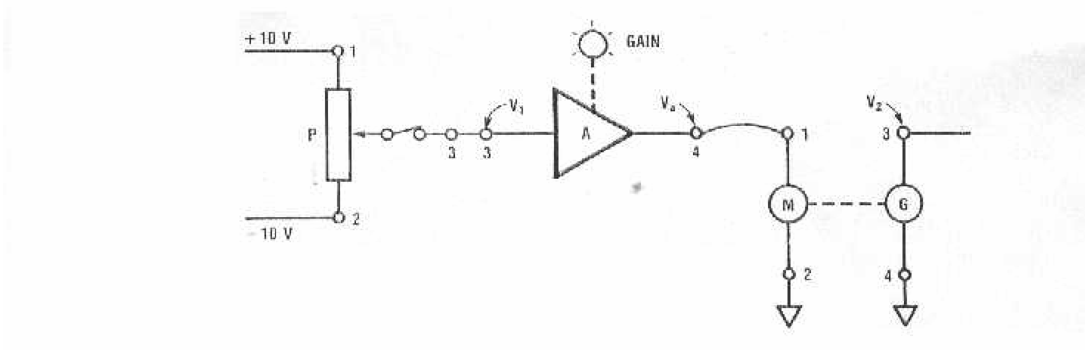


Figure 5-1-1 *control system to vary the speed of dc motor.*

In the system shown in figure 5-1-1 V_1 is the control signal, also called input signal. Voltage V_2 is the output signal, which is a direct measure of motor speed. It is also a direct measure of the geared-down shaft speed. The ratio between the control signal and the output signal is called the open-loop gain G of the system.

1.4- Methods

1- a) Connect the dc motor, power amplifier and potentiometer as shown in the schematic circuit diagram of figure 5-1-1

b) Select the dc amplifier made and set the amplifier gain equal to 1.

c) vary the potentiometer setting to obtain different values of V_1. Measure the corresponding values of generator output V_2 and record our results in Table 5-1-1.

d) Using these test results, plot V_2 versus V_1. This should give a straight line, indicating that the relationship between V_2 and V_1 is linear.

This is to be expected, because the gain calculated in table 5-1-1 is reasonably constant as V_1 is varied.

Table: 5-1-1*: measurement result of V_1,V_2 and gain Calculation by setting amplifier gain 1*

V_1	V_2	Gain = V_2 / V_1
V	V	-
8	6.20	0.776
6	4.40	0.739
4	2.70	0.675
2	1.00	0.500
0	0	0
-2	-1.00	0.500
-4	-2.71	0.675
-6	-4.40	0.7416
-8	-6.20	0.776

2- a) Using the circuit of fig. 5-1-1 with an amplifier gain of 5, vary control signal V_1 and record the corresponding value of V_2 in table 5-1-2 remember that V_2 represents the motor speed.

Table 5-1-2: *V_1, V_2 measurement results and gain calculation by setting amplifier gain =5*

V_1	V_2	G= V_2 / V_1
V	V	-
8.0	9.85	1.23
6,0	9.84	1.64
4.0	9.83	2.45
3.0	9.82	3.27
2.0	7.30	3.65
1.0	3.30	3.36
0.5	1.25	3.5
0	0	0
-0.5	-1.15	2.3
-1.0	-3.30	3.3
-2.0	-7.30	3.65
-3.0	-9.82	3.27
-4.0	-9.83	2.45
-6.0	-9.84	1.64

-8.0	-9.85	1.23

```
To get started, type one of these commands: helpwin, helpdesk, or demo.
For information on all of the MathWorks products, type tour.
» V1=[8 6 4 2 0 -2 -4 -6 -8], % control signal
V1 =
    8    6    4    2    0   -2   -4   -6   -8
»V2=[6.2 4.4 2.7 1.00 0 -1.00 -2.7 -4.4 -6.2], % value of generator outtput with a gain of 1
V2 =
Columns 1 through 7
   6.2000    4.4000    2.7000    1.0000         0   -1.0000   -2.7000
 Columns 8 through 9
  -4.4000   -6.2000
» plot(V1,V2),title('Input-output characteristic with a gain of 1')
```

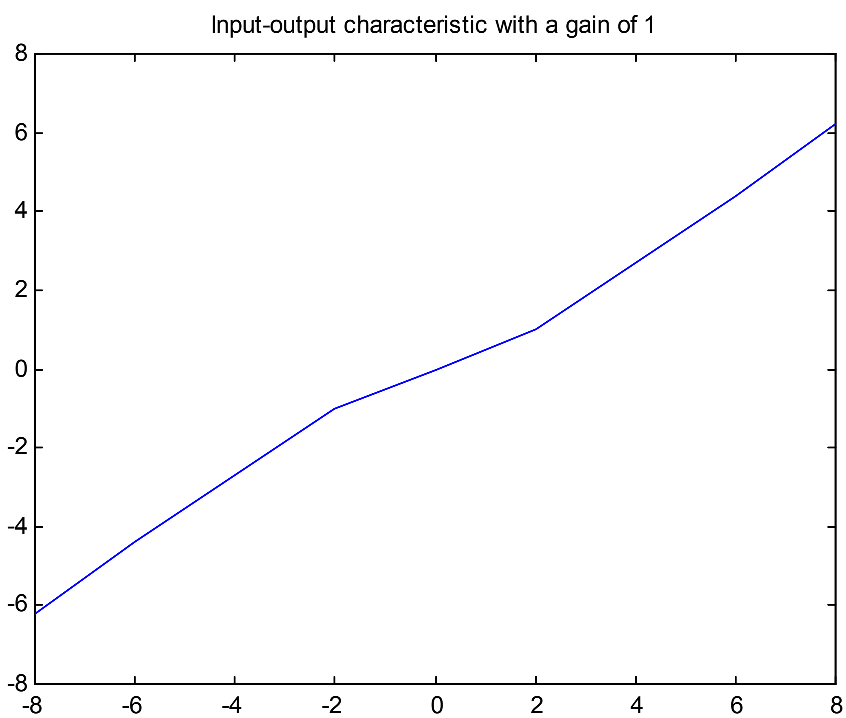


Figure 5.1.1. *Input-output characteristic with a gain of (1)*

To get started, type one of these commands: helpwin, helpdesk, or demo. For information on all of the MathWorks products, type tour.

```
» V1=[8 6 4 3 2 1 0.5 0 -0.5 -1 -2 -3 -4 -6 -8], % control signal
V1 =
 Columns 1 through 7
   8.0000    6.0000    4.0000    3.0000    2.0000    1.0000    0.5000
  Columns 8 through 14
 0   -0.5000   -1.0000   -2.0000   -3.0000   -4.0000   -6.0000
  Column 15
   -8.0000
» V2=[9.85 9.84 9.83 9.82 7.3 3.3 1.25 0 -1.25 -3.3 -7.3 -9.82 -9.83 -9.8
-9.85], % value of generator with a gain of 5
V2 =
Columns 1 through 7
    9.8500    9.8400    9.8300    9.8200    7.3000    3.3000    1.2500
Columns 8 through 14
        0   -1.2500   -3.3000   -7.3000   -9.8200   -9.8300   -9.8400
 Column 15
   -9.8500
» plot(V1,V2),title('Input-output characteristic with a gain of 5')
```

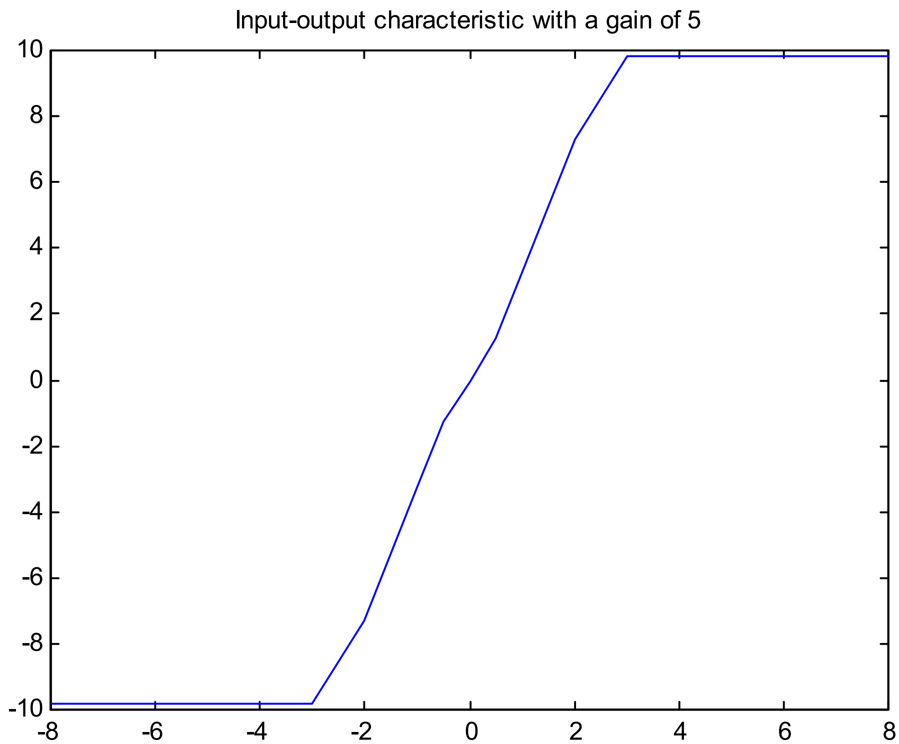


Figure 5.1.2. *Input-output characteristic with a gain of (5).*

2- Open – loop Speed Control of an AC Motor

2.1- Objectives

* To control the speed of small ac motor.

* To determine the open-loop voltage gain of an ac system.

* To observe saturation effects when ac signals are used.

When we have completed this example, we will understand the basic principles relating to ac motor control.

2.2- Equipment Required

Description

Model

Enclosure / Power Supply

8846

Connection Leads and Accessories

8944

Potentiometer

9036

Power Amplifier / Phase Shifter

9039

AC Motor / Generator

9319

Multimeter _

2.3- Discussion of Fundamental

figure 5-2-1 show the control circuit used to vary the speed of an ac motor. Potentiometer P is connected across the AC reference supply and control voltage V_1 can be varied in amplitude by moving wiper 3. The phase shifter makes V_3 equal to V_1, but displaced from it by 90. Amplifier input voltage V_3 produces a higher voltage E_b which acts on the motor control of winding. The ratio E_b / V_1 depends on the setting of the power amplifier gain. The motor reference winding is permanently connected to the 18 V, ac source. The ac tachometer (GEN) produces an output voltage V_2 which is directly proportional to the speed of the motor. The voltage gain of the system is then given by $G = V_2 / V_1$. Voltage V_1 and V_2 can be measured by means of an ac voltmeter.

2.4- Methods

1- a) Connect the system as shown in figure 5-2-1 using the ac motor module.

b) Set the amplifier gain equal to 1. Set the amplifier to the dc mode.

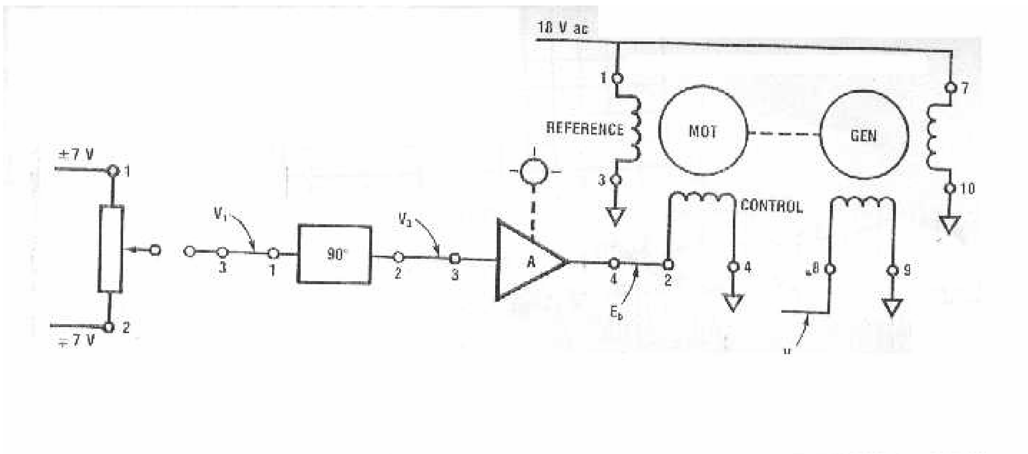


Fig. 5-2-1 *control circuit used to vary the speed of an AC motor*

Table 5-2-1: *V_1, V_2 measurement and G calculate by setting amplifier gain = 1*

V_1	V_2	G	Phase of V_2
V	V	-	DEG
6	3.30	0.55	90
5	3.12	0.624	90
4	2.99	0.722	90
3	2.55	0.85	90
2	2.02	1.07	90
1	1.08	1.08	90
0	0	0	90
0	1.08	1.08	90
2	2.02	1.07	90
3	2.55	0.85	90
4	2.99	0.722	90
5	3.12	0.624	90
6	3.33	0.55	90

Table 5-2-2: *V_1, V_2 measurement and calculate G by setting amplifier G = 5*

V_1	V_2	G

V	V	-
6.0	3.16	0.526
4.0	3.20	0.8
2.0	3.26	1.63
1.5	3.15	2.1
1.0	2.89	2.89
0.5	2.18	4.36
0	0	0
0.5	2.14	4.28
1.0	2.88	2.88
1.5	3.22	2.146
2.0	3.40	1.17
4,0	3.40	0.85
6.0	3.36	5.6

c) Vary V_1 and note the corresponding values of V_2.Record our results in table 5-2-1 and calculate the system gain G. Plot the values of V_2 versus V_1.

2- a) Set the amplifier gain equal to 5, and repeat procedure step 1 c.

b) Record the values of V_1 and V_2 in Table 5-2-2.

c) Plot graph of V_2 versus V_1, again.]

```
To get started, type one of these commands: helpwin, helpdesk, or demo
» V1=[6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6] % control signal
V1 =
Columns 1 through 12
6    5    4    3    2    1    0   -1   -2   -3   -4   -5
Column 13
-6
» V2=[3.3 3.12 2.99 2.55 2.02 1.08 0 -1.08 -2.02 -2.55 -2.99 -3.12 -3.3], %generator output
V2 =
Columns 1 through 7
3.3000    3.1200    2.9900    2.5500    2.0200    1.0800         0
```

Columns 8 through 13

-1.0800   -2.0200   -2.5500   -2.9900   -3.1200   -3.3000

» plot(V1,V2),title('Input-output characteristic with again of 1')

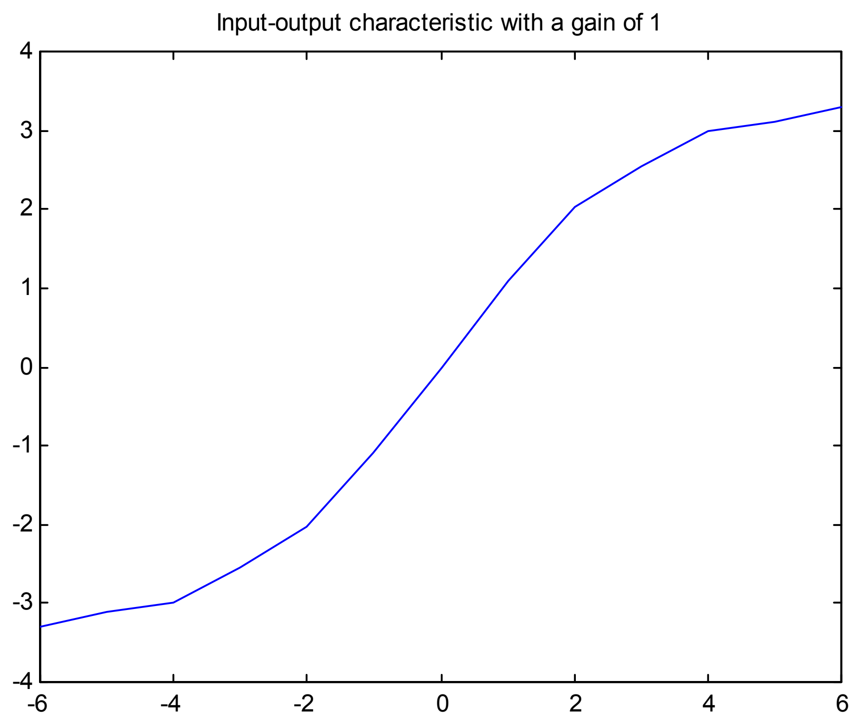


Figure 5.2.1 *input-output characteristic with a gain equal to 1*

To get started, type one of these commands: helpwin, helpdesk, or demo

» V1=[6 4 2 1.5 1.00 0.5 0 -0.5 -1.00 -1.5 -2 -4 -6], % control signal

V1 =

Columns 1 through 7

6.0000 4.0000 2.0000 1.5000 1.0000 0.5000 0

Columns 8 through 13

-0.5000   -1.0000   -1.5000   -2.0000   -4.0000   -6.0000

» V2=[3.16 3.2 3.26 3.15 2.89 2.18 0 -2.14 -2.89 -3.22 -3.4 -3.4 -3.36], % value of generator output with gain of 5

V2 =

Columns 1 through 7

3.1600 3.2000 3.2600 3.1500 2.8900 2.1800 0

Columns 8 through 13

-2.1400   -2.8900   -3.2200   -3.4000   -3.4000   -3.3600

» Plot (V1, V2), title ('Input-output characteristic with gain of 5')

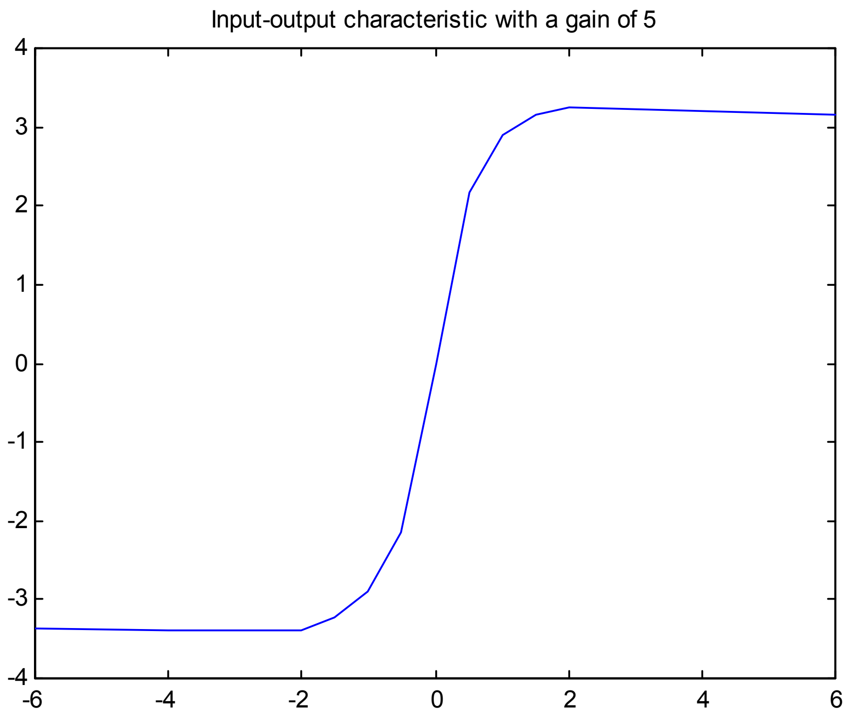


Figure 5.2.2 *input-output characteristic of a gain equal 5*

3- Closed-loop Speed Control of an AC Motor

3-1. Objectives

* To observe the closed-loop behavior of an ac motor when its speed is controlled by means of a variable dc signal. When we have completed this example, we will be able to demonstrate that closed-loop control makes the voltage-speed characteristic of an ac motor more linear than in open loop. We will also see a practical application of a modulator. Before proceeding with this example.

3-2. Equipment Required

Description

Model

Enclosure / Power Supply
8846

Connection Leads and Accessories
8944

Potentiometer
9036

Error detector
9037

Modulator
9038

Power Amplifier / Phase Shifter
9039

Ac Motor / Generator
9019

Multimeter -

3-3. Discussion of Fundamentals

It is identical to the circuit we used in example 2, except that a dc instead of an ac control signal Vc is now used. An error detector has also been added. Modulator M converts the dc signal Vc into an ac signal Vc. The open loop gain is given by G = V_2 / Vc. Note that this gain is expressed as the ratio of an ac signal V_2 to a dc signal Vc. Up till now, the gain of a system was always expressed as the ratio of two dc signals or of two ac signals. After we have found the relationship between V_2 and Vc in open loop, we will feed the V_2 signal back to the input. This feedback connection is shown by the dotted line in Figure 5-3-1. we will find the resulting closed-loop relationship between V_2 and Vc is much more linear. This is because the closed-loop gain H is more constant than the open loop gain G. We will recall that H = V_2 / Vc.

3-4. Methods

1- a) Connect the open-loop circuit as shown in Figure 5-3-1 Do not make the dotted feedback connection.

b) Select the ac amplifier mode and adjust the amplifier gain to 5.

c) Vary the potentiometer setting and record the values of Vc and V_2 in Table 5-3-1. Calculate the open-loop gain for each setting.

2- a) Close the feedback loop by making the dotted connection in figure 5-3-1. Vary the potentiometer setting and note that the direction of rotation can again be either forward or reverse. We should also note that it take much larger control voltage Vc to obtain given speed. The reason is that the control signal is now equal to the sum of the input signal V_1 with V_2.

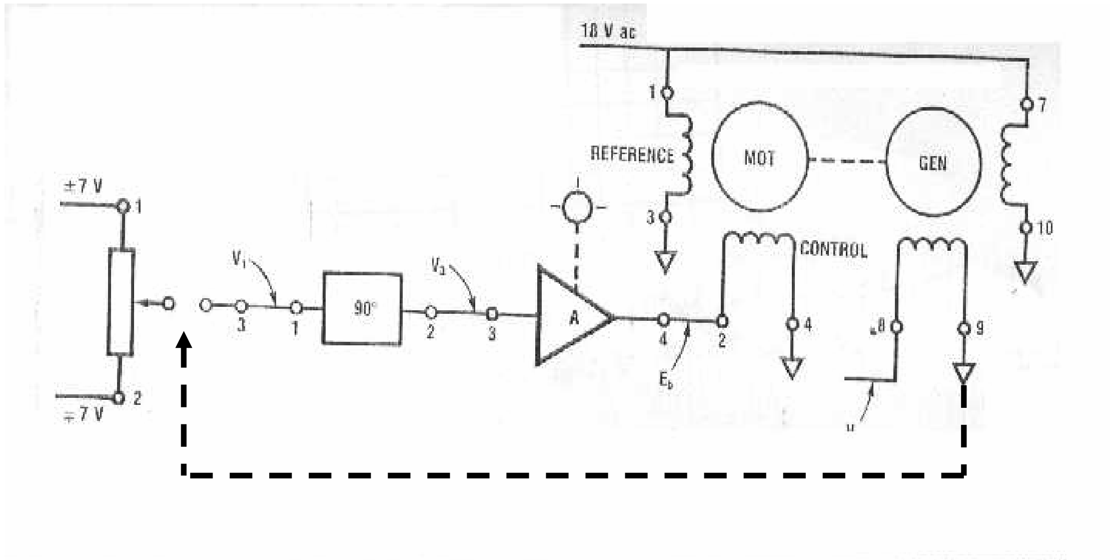


Fig. 5-3-1 *closed-loop control of an AC Motor*

Table 5-3-1 *V_c , V_2 Value and calculate The open-loop gain to each setting.*

Vc	V_2	G= V_2 / Vc	Speed
V	**V**	**-**	**r/min**
2.0	3.32	1.66	200
1.0	2.97	3.97	175
0.5	2.15	4.30	115
0	0	0	0
-0.5	2.19	-4.33	-115
-1.0	3.03	-3.03	-175
-2.0	3.46	-1.78	-200

Table 5-3-2 *V_1 , V_2 value and calculate the closed-loop gain H in each case*

V_c	V_2	H= V_2 / V_c	Speed
V	**V**	**-**	**r/ min**
8	3.30	0.41	170
6	3.27	0.595	167
4	2.82	0.75	163
3	1.65	0.825	92
0	0	0	0
-2	1.7	0.85	-90
-4	2.8	0.75	-169
-6	3.27	0.45	-167
-8	3.3	0.41	-170

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

```
» Vc = [2 1 0.5 0 -0.5 -1 -2], % control signal
Vc =
   2.0000 1.0000 0.5000 0   -0.5000   -1.0000   -2.0000

» Speed = [200 175 115 0 -115 -175 -200], % speed (r/min)
Speed =
  200 175 115 0  -115  -175  -200
» Plot (Vc, Speed), title ('Curve of speed verses Vc')
```

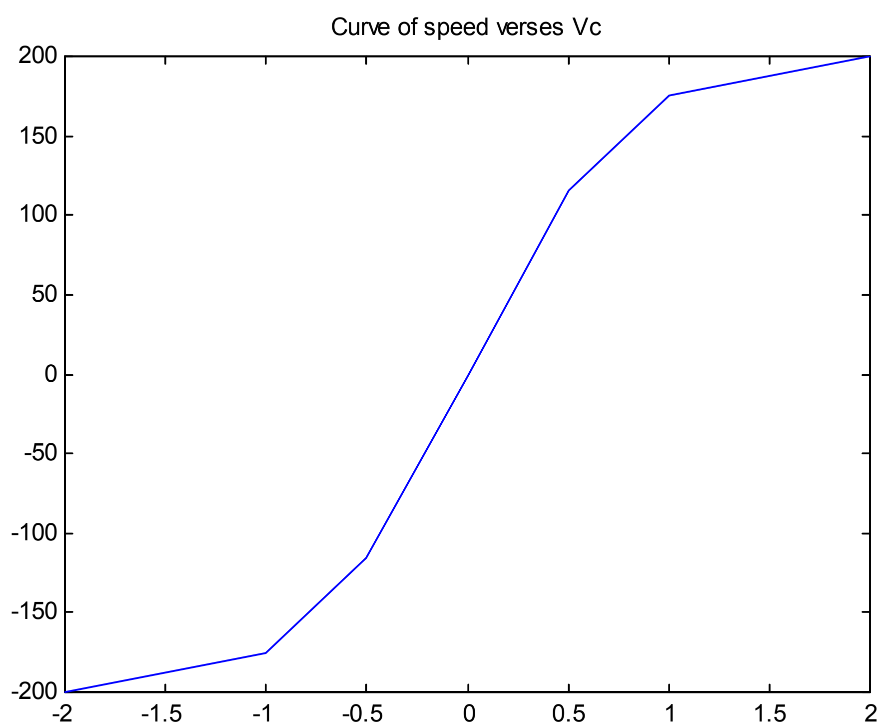


Figure 5.3.1-*curve value of speed versus V_2*

To get started, type one of these commands: helpwin, helpdesk, or demo.
For information on all of the MathWorks products, type tour.

```
» Vc= [8 6 4 2 0 -2 -4 -6 -8], % control signal
Vc =
    8 6 4 2 0    -2    -4    -6    -8

» Speed = [170 167 163 92 0 -92 -163 -167 -170], % speed (r/min)
Speed =
   170 167 163 92 0  -92  -163  -167  -170

» Plot (Vc, Speed), title ('Curve of speed verses Vc')
```

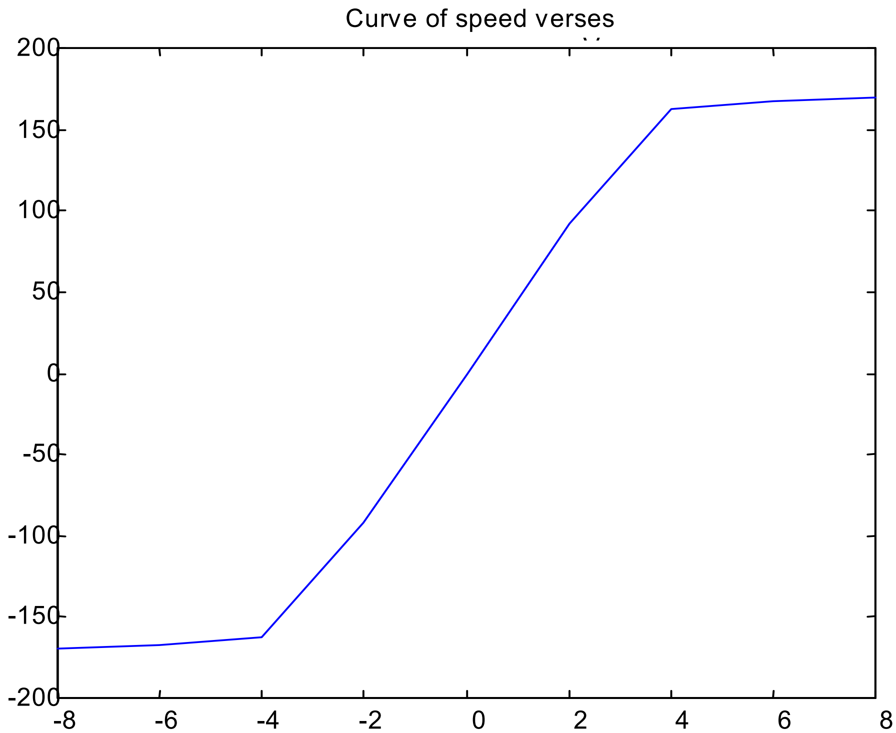


Figure 5.3.2-curve *value of speed versus Vc*

4 ON – OF CONTROL OF THE LEVEL

ctives- -To understand the operation of a closed-loop ON-OF control system.

-To understand the effects ofhysteresis on the control.

equisites - Knowledge of the operation of a closed-loop control system.

◊ **Methodology** - Guided example.

◊ **List of Equipment** - DL 2314.

- Digital multimeter.

- Chronometer.

- Set of leads.

► Delivery Valve fully open (turn the knob counter-clockwise).

► Motor Valve fully open (angular rotation = 0).

► Sol Valve open (ON) using the interface ON-OF DRIVER.

► Man Valve fully open (turn the knob counter-clockwise)

► Drain Valve fully open (turn the knob-clockwise)

► Needle Valve fully open (turn the knob clock-wise)

► Air Valve fully open (turn the knob counter-clockwise)

► Level of the water in the process tank: 8 cm.

► Set point 1 Knob 0V

► Hysteresis Knob 0%

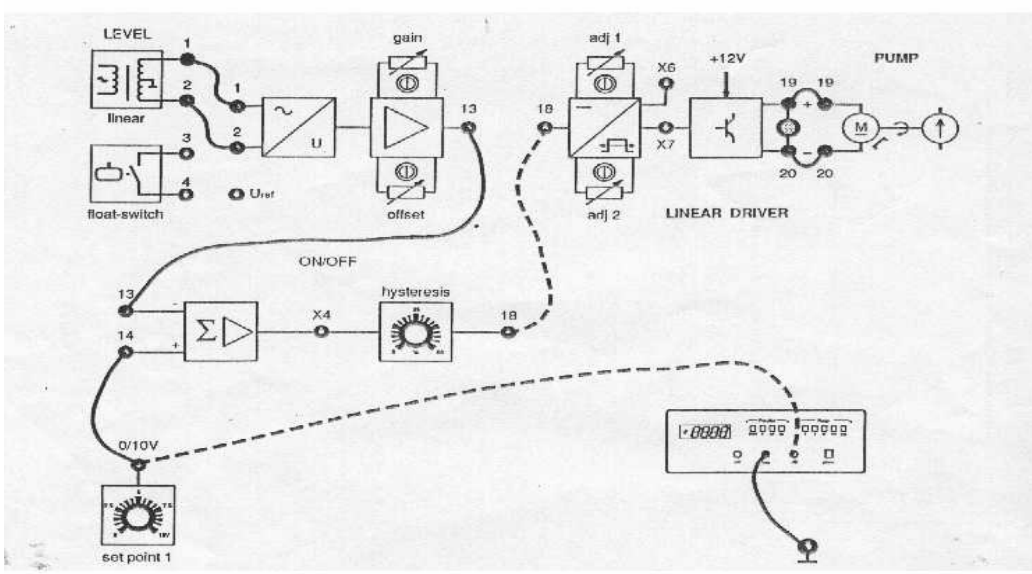


Fig. 5-4-1 *ON-OFF control*

Table 5-4-1: *measure of up and down times of level between start and stop of the pump with an hysteresis of 0%, 15% and 30%*

Hysteresis %	**0**	**15**	**30**
Set point (cm)	12.1	12.1	12.1
Lower limit set point (cm)	11.9	11.1	10.5
Up rising time of the level (sec)	3.25	38.4	29.3
Upper limit set time (cm)	12.1	12.7	13.5
Lowing time of the level (sec)	1.34	14.8	103.75

To get started, type one of these commands: helpwin, helpdesk, or demo.
For information on all of the MathWorks products, type tour.

```
» t=[0 1.34 3.25], % Lowering time & Up-raising time (sec)
t =
      0 1.3400 3.2500

» L=[0 11.9 12.1], % Lower & Upper limit set point
L =
      0 11.9000 12.1000

» Plot (t, L), title ('Level between start & stop pump with an hysteresis of 0%')
```

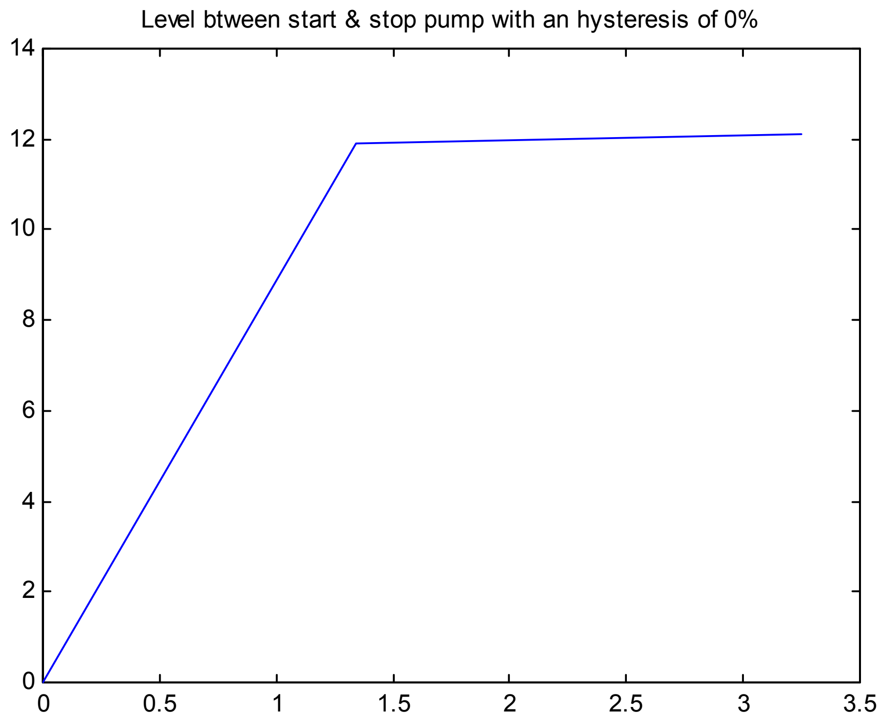


Figure 5.4.1 *the characteristic diagram of the hysterics 0%*

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

```
» t=[0 14.8 38.4], % Lowering & Upper-raising time (sec)
t =
       0 14.8000 38.4000

» L=[0 11.1 12.1], % Lower & Upper limit set point (cm)
L =
       0   11.1000   12.1000

» Plot (t, L), title ('Level between start & stop pump with an hysteresis of 15%')
```

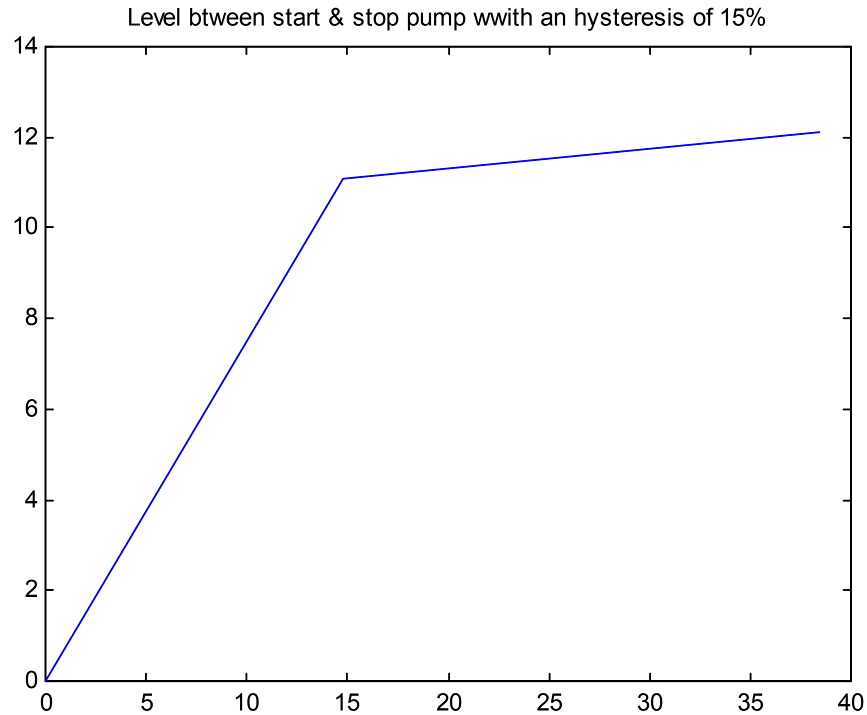


Figure 5.4.2 *the characteristic diagram of the hysterics 15%*

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

» t=[0 103.75 29.35], % Lowering & Upper-raising time (sec)

t =

0 103.7500 29.35000

» L=[0 10.5 12.1], % Lower & Upper limit set point(cm)

L =

0 10.5000 12.1000

» Plot (t, L), title ('Level between start & stop pump with an hysteresis of 30%')

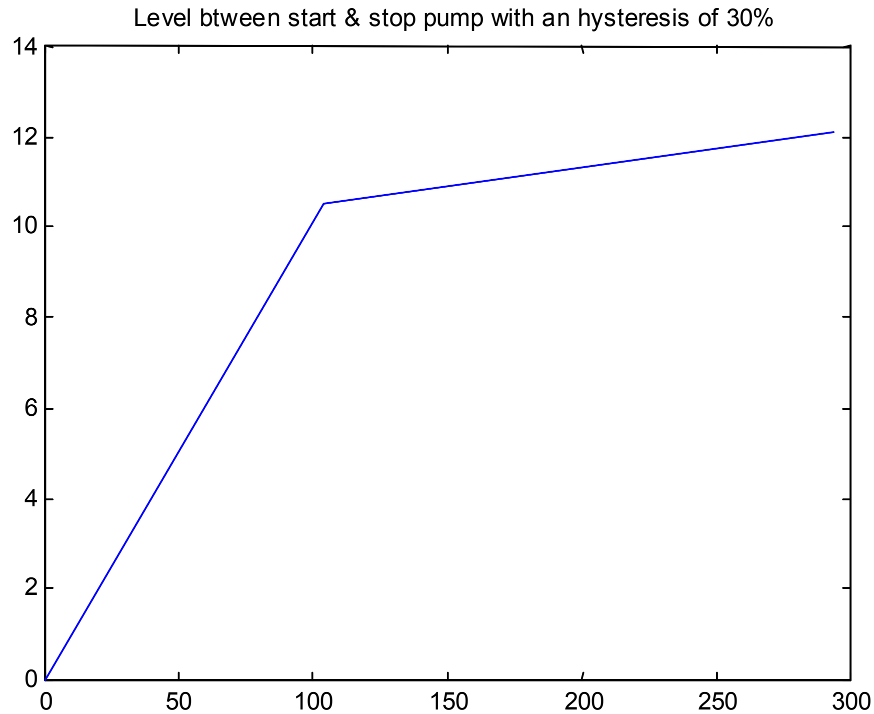


Figure 5.4.3 the characteristic diagram of the hysterics 30%

5- Discussion of examples

We concluded from the examples of single-input, single-output, all elements are independent of the control signal u (t). We demonstrate that on examples 1and 2 the open-loop control systems, the saturation point can be seen only by increasing the amplifier gain. We are comparing the results of examples 2 and 3 open-loop speed control of an ac motor, and closed-loop speed control of an ac motor. We will be able to demonstrate that, closed-loop control system makes the voltage-speed characteristic of an ac motor more linear than open loop control system. We seen in example 4, ON-OF level, from results curves characteristic, the control system will be improved by increasing hyteresis value at 0%-30%

Finally, we look only for single-input, single-output control systems in this study, because we did not find the multiple-input, multiple-output control systems

Chapter VI-Conclusion

The controllability matrix (S), and observability matrix (V) are achieved the present practice of controllable/ observable control systems

We have demonstrated in this study, that the determinant of system matrix (A) is shown to affect the design of controllable/ observable control systems. In chapter IV, we have used five-order system matrix with different values, zero, positive number, and negative number. We concluded from that, if the determinant of system matrix is positive or negative number, the system is fully controllable by any input signal connection, and we can improve controllability by adding another input (multiple-input). When the system matrix determinant is zero, it can be uncontrollable by many input-signal. This situation can be changed to controllability by adding another input-signal. Also when the system matrix determinant is positive or negative number, the system is fully observable by any output connection, and we can improve observability of system by adding another output (multiple-output). When the system matrix determinant is zero, the system can be unobservable by many outputs. This situation can be changed to observability by adding another output. We advice designer of controllable/ observable control system, to look carefully for the system, and its matrix before starting his design. We have concluded that, the feedback gains are improved the design of controllable control system. In other words, the feedback gains and controllable control system are related. The feedback gains and observable control system are not related. In other words, the feedback gains destroyed observability control system.

Appendix A:

System matrix second order

```
  To get started, type one of these commands: helpwin, helpdesk, or demo.
  For information on all of the MathWorks products, type tour.
» A=[0 1;-4 -5], % System matrix second orde
A =
    0    1
   -4   -5
» B=[0;1], % Input matrix
B =
    0
    1
» G=Sym('[s 0;0 s]')
G =
[ s, 0]
[ 0, s]
» G-A
ans =
```

```
[   s,  -1]
[   4, s+5]
» det(G-A)
ans =
s^2+5*s+4
» P=[1 5 4]
P =
     1     5     4
» r=roots(P)
r =
    -4
    -1
» S=[B A*B], % Controllability matrix
S =
     0     1
     1    -5
» det(S)
ans =
    -1
```

Appendix B

Controllability of system matrix fifth order det (A) =0

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

```
» A=[2 0 1 0 2;2 0 1 0 2;1 1 0 0 2;0 2 0 1 1;1 1 3 0 0], % System matrix fifth order
A =
    2    0    1    0    2
    2    0    1    0    2
    1    1    0    0    2
    0    2    0    1    1
    1    1    3    0    0
» det(A)
ans =
    0
» B=[1;0;0;0;0], %
B =
    1
    0
    0
    0
    0
» G=Sym ('[s 0 0 0 0;0 s 0 0 0;0 0 s 0 0;0 0 0 s 0;0 0 0 0 s]')
 G =
 [ s, 0, 0, 0, 0]
[ 0, s, 0, 0, 0]
```

```
[ 0, 0, s, 0, 0]
[ 0, 0, 0, s, 0]
[ 0, 0, 0, 0, s]
» G-A
 ans  [ s-2,   0,  -1,   0, -2]
[  -2,   s,  -1,   0,  -2]
[  -1,  -1,   s,   0,  -2]
[   0,  -2,   0, s-1,  -1]
[  -1,  -1,  -3,   0,    s]
» det(G-A)
 ans =
 s^5-3*s^4+8*s^2-10*s^3+4*s
» P=[1 -3 -14 8 8 0]
P =
    1    -3   -14     8     8     0
» r=roots(P)
r =
         0
    5.3022
   -2.7544
   -0.5478
    1.0000
» S=[B A*B A^2*B A^3*B A^4*B], % Controllability matrix
S =
     1     2     7    34   160
     0     2     7    34   160
     0     1     6    28   132
     0     0     5    26   126
     0     1     7    32   152
» det(S)
ans =
    215689
```

Appendix C

Controllability of system matrix fifth order det (A)= 225

To get started, type one of these commands: help win, helpdesk, or demo.

For information on all of the Math Works products, type tour.

```
» A=[5 0 2 0 3;0 1 2 3 0;4 2 1 0 1;1 -4 -1 0 3;0 -1 3 -2 -2], % System matrix fifth order
A =
     5     0     2     0     3
     0     1     2     3     0
     4     2     1     0     1
     1    -4    -1     0     3
     0    -1     3    -2    -2
» det(A)
ans =
   225
» B=[0;1;0;0;0]
B =
     0
     1
     0
     0
     0
» G=Sym('[s 0 0 0 0;0 s 0 0 0;0 0 s 0 0;0 0 0 s 0;0 0 0 0 s]')
```

```
G =
[ s, 0, 0, 0, 0]
[ 0, s, 0, 0, 0]
[ 0, 0, s, 0, 0]
[ 0, 0, 0, s, 0]
[ 0, 0, 0, 0, s]
» G-A

ans =
[ s-5,   0,  -2,   0,  -3]
[   0, s-1,  -2,  -3,   0]
[  -4,  -2, s-1,   0,  -1]
[  -1,   4,   1,   s,  -3]
[   0,   1,  -3,   2, s+2]

» det(G-A)
ans =
s^5-66*s^2-309*s-5*s^4-225
» P=[1 -5 0 -66 -225]
P =
     1    -5     0   -66  -225
» r=roots(P)
r =
   7.0018
   0.1077 + 3.8055i
   0.1077 - 3.8055i
  -2.2172
```

```
» S=[B A*B A^2*B A^3*B A^4*B]
S =
     0     0     1    56   308
     1     1    -7   -28   201
     0     2     3     8   180
     0    -4    -9    71   172
     0    -1    15     4   -98
» det(S)
ans =
       71492
```

Controllability of system matrix fifth order det (A)= -209

To get started, type one of these commands: helpwin, helpdesk, or demo.
For information on all of the MathWorks products, type tour.

```
]» A=[-5 0 2 0 3;0 1 2 -3 0;-4 2 1 0 1;1 4 -1 0 3;0 -1 3 -2 2], % System matrix fifth order
A =
   -5    0    2    0    3
    0    1    2   -3    0
   -4    2    1    0    1
    1    4   -1    0    3
    0   -1    3   -2    2
» det(A)
ans =
  -209
» B=[0;0;1;0;0]
B =
    0
    0
    1
    0
    0
» G=Sym('[s 0 0 0 0;0 s 0 0 0;0 0 s 0 0;0 0 0 s o;0 0 0 0 s]')


G =
[ s, 0, 0, 0, 0]
[ 0, s, 0, 0, 0]
[ 0, 0, s, 0, 0]
```

```
[ 0, 0, 0, s, o]
[ 0, 0, 0, 0, s]
» G-A
ans =
 [ s+5,   0,  -2,   0,  -3]
 [   0, s-1,  -2,   3,    0]
 [   4,  -2, s-1,   0,   -1]
 [  -1,  -4,   1,   s,  o-3]
 [   0,   1,  -3,   2,  s-2]
» det(A)
ans =
  -209
» det(G-A)
ans =
s^5+s^4-2*s^3*o+4*s^3-3*s^2*o+44*s^2+4*s*o-85*s-35*o+209
» P=[1 14 44 -85 209]
P =
     1    14    44   -85   209
» r=roots(P)
r =
  -7.9335 + 2.7794i
  -7.9335 - 2.7794i
   0.9335 + 1.4444i
   0.9335 - 1.4444i
» P=[1 1 4 44 -85 209]
P =
     1     1     4    44   -85   209
» r=roots(P)
r =
```

```
  -4.2478
   0.4070 + 3.3752i
   0.4070 - 3.3752i
   1.2169 + 1.6663i
   1.2169 - 1.6663i
» S=[B A*B A^2*B A^3*B A^4*B]

S =
     0     2     1    22   -147
     0     2     7   -47   -177
     1     1     0    19   -188
     0    -1    18    56   -260
     0     3     9   -25    -58

» det(S)
ans =
      753099
```

Appendix E

Observability of system second order

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

```
» A=[0 1;-4 -5], % System matrix second order
A =
```

```
     0     1
    -4    -5
» C=[0 1], % Ouypot mtrix
C =
     0     1
» G=Sym('[s 0;0 s]')
G =
[ s, 0]
[ 0, s]
 » G-A
ans =
[   s,  -1]
[   4, s+5]
» det(G-A)
ans =
s^2+5*s+4
» P=[1 5 4]
P =
     1     5     4
» r=roots(P)
r =
    -4
    -1
» V=[C;C*A]
V =
     0     1
    -4    -5
» det(V)
ans =
```

4

Appendix F

Observability of system fifth order det (A) = 0

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

» A=[2 0 1 0 2;2 0 1 0 2;1 1 0 0 2;8 2 0 1 1;3 1 3 0 0], % System matrix fifth order

A =

```
    2     0     1     0     2
    2     0     1     0     2
    1     1     0     0     2
    8     2     0     1     1
```

```
    3     1     3     0     0
» det(A)
ans =
    0
» C=[1 0 0 0 0], % Output matrix
C =
    1     0     0     0     0
» G=Sym('[s 0 0 0 0;0 s 0 0 0;0 0 s 0 0;0 0 0 s 0;0 0 0 0 s]')
G =
[ s, 0, 0, 0, 0]
[ 0, s, 0, 0, 0]
[ 0, 0, s, 0, 0]
[ 0, 0, 0, s, 0]
[ 0, 0, 0, 0, s]
» G-A
ans =
[ s-2,   0,  -1,   0,  -2]
[  -2,   s,  -1,   0,  -2]
[  -1,  -1,   s,   0,  -2]
[  -8,  -2,   0, s-1,  -1]
[  -3,  -1,  -3,   0,   s]
» det(G-A)
ans =
s^5-3*s^4+8*s^2-14*s^3+8*s
» P=[1 -3 -14 8 8 0]
P =
    1    -3   -14     8     8     0
» r=roots(P)
r =
```

```
      0
   5.3022
  -2.7544
  -0.5478
   1.0000
» V=[C;C*A;C*A^2;C*A^3;C*A^4], % Observability matrix
V =
    1    0    0    0    0
    2    0    1    0    2
   11    3    8    0    6
   54   14   32    0   44
  300   76  200    0  200
» det(V)
ans =    0
```

Appendix G:

Observability of system matrix fifth order det (A) = 225

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type to

```
» A=[5 0 2 0 3;0 1 2 3 0;4 2 1 0 1;1 -4 -1 0 3;0 -1 3 -2 -2]
A =
    5    0    2    0    3
    0    1    2    3    0
    4    2    1    0    1
    1   -4   -1    0    3
    0   -1    3   -2   -2
» det(A)
ans =
  225
```

```
» C=[0 1 0 0 0]
C =
     0     1     0     0     0
» G=Sym('[s 0 0 0 0;0 s 0 0 0;0 0 s 0 0;0 0 0 s 0;0 0 0 0 s]')
G =
[ s, 0, 0, 0, 0]
[ 0, s, 0, 0, 0]
[ 0, 0, s, 0, 0]
[ 0, 0, 0, s, 0]
[ 0, 0, 0, 0, s]
» G-A
ans =
[ s-5,   0,  -2,   0,  -3]
[   0, s-1,  -2,  -3,   0]
[  -4,  -2, s-1,   0,  -1]
[  -1,   4,   1,   s,  -3]
[   0,   1,  -3,   2, s+2]
» det(G-A)
ans =
s^5-66*s^2-309*s-5*s^4-225
» P=[1 -5 0 -66 -309 -225]
P =
     1    -5     0   -66  -309  -225
» r=roots(P)
r =
   7.1913
   0.4061 + 4.0137i
   0.4061 - 4.0137i
  -2.0786
```

```
 -0.9249
» V=[C;C*A;C*A^2;C*A^3;C*A^4]
V =
     0     1     0     0     0
     0     1     2     3     0
    11    -7     1     3    11
    62   -28    39   -43    21
   423   201   213  -126    54
» det(V)
ans =
      438741
```

Appendix H

Observability of system matrix fifth order det (A) = -209

```
  To get started, type one of these commands: helpwin, helpdesk, or
demo.
  For information on all of the MathWorks products, type tour.
 » A=[-5 0 2 0 3;0 1 2 -3 0;-4 2 1 0 1;1 4 -1 0 3;0 -1 3 -2 2]
A =
    -5     0     2     0     3
     0     1     2    -3     0
    -4     2     1     0     1
     1     4    -1     0     3
     0    -1     3    -2     2
» det(A)
ans =
  -209
» C=[0 0 1 0 0]
C =
     0     0     1     0     0
```

```
» G=Sym('[s 0 0 0 0;0 s 0 0 0;0 0 s 0 0;0 0 0 s 0;0 0 0 0 s]')
G =
[ s, 0, 0, 0, 0]
[ 0, s, 0, 0, 0]
[ 0, 0, s, 0, 0]
[ 0, 0, 0, s, 0]
[ 0, 0, 0, 0, s]
» G-A
ans =
[ s+5,   0,  -2,   0,  -3]
[   0, s-1,  -2,   3,   0]
[   4,  -2, s-1,   0,  -1]
[  -1,  -4,   1,   s,  -3]
[   0,   1,  -3,   2, s-2]
» det(G-A)
ans =
s^5+s^4+4*s^3+44*s^2-85*s+209
» P=[1 1 4 44 -85 209]
P =
     1     1     4    44   -85   209
» r=roots(P)
r =
  -4.2478
   0.4070 + 3.3752i
   0.4070 - 3.3752i
   1.2169 + 1.6663i
   1.2169 - 1.6663i
» V=[C;C*A;C*A^2;C*A^3;C*A^4]
V =
```

```
     0     0     1     0     0
    -4     2     1     0     1
    16     3     0    -8    -9
   -88   -20    19     9     6
   373    48  -188    48  -206
» det(V)
ans =
     -443769
```

Appendix I

Feedback affect of design of controllable/ observable control system

To get started, type one of these commands: helpwin, helpdesk, or demo.

For information on all of the MathWorks products, type tour.

```
» A=[1 2 1;2 3 1;3 1 2]
A =
     1     2     1
     2     3     1
     3     1     2
» det(A)
ans =
    -4
» B=[0;0;1]
B =
     0
     0
     1
» C=[0 0 1]
```

```
C =

     0     0     1

» S=[B A*B A^2*B]

S =

     0     1     5
     0     1     7
     1     2     8

V=[C;C*A;C*A^2]

V =

     0     0     1
     3     1     2
    11    11     8

» det(V)

ans =

    22

» Q=Sym('[k1 k2 k3]')

Q =

[ k1, k2, k3]

» S=[B (A-B*Q)*B (A-B*Q)^2*B]

S =

[            0,            1,                 5-k3]
[            0,            1,                 7-k3]
[            1,         2-k3, 4-k1-k2+(2-k3)^2]

 » G=Sym('[s 0 0;0 s 0;0 0 s]')

G =

[ s, 0, 0]
[ 0, s, 0]
[ 0, 0, s]

 » G-A
```

```
ans =
[ s-1,  -2,  -1]
[  -2, s-3,  -1]
[  -3,  -1, s-2]
 » det(G-A)
ans =
s^3-6*s^2+3*s+4
 » G-A+B*Q
ans =
[    s-1,     -2,     -1]
[     -2,    s-3,     -1]
[  -3+k1,  -1+k2, s-2+k3]
 » det(G-A+B*Q)
ans =
s^3-6*s^2+s^2*k3+3*s-4*s*k3+s*k2+4-k3+k2+k1*s-k1
 » K1=0
K1 =
     0
» K2=0
K2 =
     0
» K3=0
K3 =
     0
```

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